Exercise 1: Complete lattices

1. Let $M = \{a, b, c\}$. Define a relation $R$ such that $(M, R)$ is a complete lattice.

2. For a totally ordered set $S$, $(\mathcal{P}(S), \subseteq)$ is a complete lattice. Define another relation $R$ such that $(\mathcal{P}(S), R)$ is a complete lattice.

3. Is $(\mathbb{R}, \leq)$ a complete lattice? If not, how can you extend $\mathbb{R}$ such that it becomes a complete lattice?

4. Let $\mid$ be the relation of divisibility, i.e. $a \mid b$ means $a$ divides $b$. Is
   - $(\mathbb{N}, \mid)$
   - $(\mathbb{N}\setminus\{0\}, \mid)$
   - $(\mathbb{N}\setminus\{0\} \cup \{\infty\}, \mid)$
   a complete lattice?

Exercise 2: Comparing different approaches

Consider the following WHILE program from the slides:

\[
\begin{align*}
[y := x;] &; \\
[z := 1]; &; \\
\text{while } [y > 0] &; do \\
[z := z \ast y]; &; \\
[y := y - 1]; &; \\
[y := 0]; &; \\
\end{align*}
\]

Let $F : (\mathcal{P}(\text{Var} \times \text{Lab}))^{12} \rightarrow (\mathcal{P}(\text{Var} \times \text{Lab}))^{12}$ be the function defined by the data flow equations (cf. slides on p. 31 ff.). Further, let $(\alpha, \gamma)$ be the Galois connection for the Reaching Definitions analysis (cf. slides on p. 69 ff.)

1. Prove that $\vec{\alpha} \circ G \circ \vec{\gamma} \subseteq F$, i.e. show that
   \[
   \alpha(G_j(\gamma(RD_1), \ldots, \gamma(RD_{12}))) \subseteq F_j(RD_1, \ldots, RD_{12})
   \]
   holds for all $j$. Here, $\vec{f}$ denotes the application of function $f$ to all entries of a tuple or vector.

2. Check whether $F = \vec{\alpha} \circ G \circ \vec{\gamma}$.

3. Prove by induction over $n$ that $(\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset) \subseteq F^n(\emptyset)$.

4. Prove that $\vec{\alpha}(G^n(\emptyset)) \subseteq (\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset)$. You may use that $\vec{\alpha}(\emptyset) = \emptyset$ and $G \subseteq G \circ \vec{\gamma} \circ \vec{\alpha}$.

Submission

- Deadline: 10.05.2010, 12:00, per mail to bieniusa@informatik.uni-freiburg.de, or on paper to Annette Bieniusa, Geb. 079, Room 000-14.
- Late submissions will not be marked.
- Do not forget to put your name on the exercise sheet.
- You might want to read up in Appendix A of Principles of Program Analysis.