Lecture: Program analysis
Exercise 6
http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2010ss/

1 Coinduction

1.1 Generating Functions

Suppose a generating function $F : \mathcal{P}(\{a, b, c\}) \rightarrow \mathcal{P}(\{a, b, c\})$ on the universe $\{a, b, c\}$ is defined by the following inference rules:

$$\frac{}{a} \qquad \frac{c}{b} \qquad \frac{a \quad b}{c}$$

1. Write out the set of pairs in the relation F explicitly.
2. List all the F-closed and F-consistent sets.
3. What are $\mathit{lfp}(F)$ and $\mathit{gfp}(F)$?

1.2 Relations

Consider a context free grammar with start symbol N and productions $N ::= \mathit{Zero} \mid \mathit{Succ}(N)$. It can be rephrased as an inductive definition:

$$\mathit{Zero} \in N \qquad \frac{n \in N}{\mathit{Succ}(n) \in N}$$

1. What set N is defined if you interpret the rules inductively? What does a coinductive interpretation yield?
2. Let us now define a relation $\leq$ on N in the following way:

   $$\mathit{Zero} \leq n \quad \forall n \in S \qquad \frac{n \leq m}{\mathit{Succ}(n) \leq \mathit{Succ}(m)}$$

   Let $R = \{(x, y) | x, y \in N : x \leq y\} \subseteq N \times N$.

   - Define the generating function $S : R \rightarrow R$ for this relation. Check that S is a monotone function.
   - Can you find a pair (x, y) such that $(x, y) \in \mathit{gfp}(S)$, but$(x, y) \notin \mathit{lfp}(S)$?
   - Prove that $\mathit{gfp}(S)$ is transitive and reflexive.

2 Control Flow Analysis

2.1 Analyzing a program by hand

Consider the following program:

let f = fn $y \Rightarrow y$ in
    let g = fn $x \Rightarrow f$ in
        let h = fn $v \Rightarrow v$ in
            g (g h)

Add labels to the program, and guess an analysis result. Use Table 3.1 in the book, p. 146, to verify that it is indeed an acceptable guess.

2.2 Enhancing the analysis

Modify the Control Flow Analysis of Table 3.1. to take account of the left to right evaluation order imposed by a call-by-value semantics: In the clause [*app*] there is no need to analyze the operand if the operator cannot produce any closures. Try to find a program where the modified analysis accepts a result which is rejected by Table 3.1.

Submission

- Deadline: 28.06.2010, 11:00, per mail to `bieniusa@informatik.uni-freiburg.de`, or on paper to Annette Bieniusa, Geb. 079, Room 000-14.
- Late submissions will not be marked.
- Do not forget to put your name on the exercise sheet.