1 Monotone Frameworks

1. Show that Constant Propagation (as defined in Sec. 2.3.3 of Nielson & Nielson and on the slides) is a Monotone Framework.

2. Show that the Reaching Definitions Analysis is a Bit Vector Framework.

Solution

1. We have to show that

   • \( L = (\mathbf{Var} \rightarrow \Sigma^\top, \sqsubseteq) \) is a complete lattice which satisfies the Ascending Chain Condition, and
   
   • \( \mathcal{F}_\text{CP} = \{ f | f \text{ is a monotone function on } \mathbf{State}_\text{CP} \} \)

   contains the identity function

   and is closed under function composition.

   As defined in chap. 2.3.3., \( L \) is by construction a complete lattice. It also satisfies ACC because \( \mathbf{Var} \) is finite for a given program. Further, the identity function is monotone, and compositions of monotone functions are again monotone.

2. We have to show that

   • \( L = (\mathcal{P}(D), \sqsubseteq) \) for a finite set \( D \) and \( \sqsubseteq \) is either \( \subseteq \) or \( \supseteq \), and
   
   • \( \mathcal{F} = \{ f : \mathcal{P}(D) \rightarrow \mathcal{P}(D) | \exists Y_1, Y_2 : \forall Y \subseteq D : f(Y) = (Y \cap Y_1) \cup Y_2 \} \).

   For the RD Analysis, we have \( L = (\mathcal{P}(\mathbf{Var} \times \mathbf{Lab}^\top), \subseteq) \), and \( \mathbf{Var} \times \mathbf{Lab}^\top \) is finite.

   Further, set \( Y_1 = D \setminus l_k \) and \( Y_2 = l_g \). Then,

   \[
   f(l) = (l \cap (D \setminus l_k)) \cup l_g \\
   = ((l \setminus l_k) \cap D) \cup l_g \\
   = (l \setminus l_k) \cup l_g
   \]

2 Detection of Signs Analysis

In a Detection of Signs Analysis, one models all negative numbers by the symbol \( - \), zero by 0, and all positive numbers by +. E.g., the set \( \{-2, -1, 1\} \) is modeled by \( \{+,-\} \).

Let \( S_* \) be a program, \( \mathbf{Var}_* \) the finite set of variables in \( S_* \). Take \( L = \mathbf{Var}_* \rightarrow \mathcal{P}(\{-+,+\}) \) and define an instance of a Monotone Framework for performing Detection of Signs Analysis.

Similarly, take \( L' = \mathbf{Var}_* \times \mathcal{P}(\{-+,+\}) \) and define an instance of a Monotone Framework for performing Detection of Signs Analysis. Is there any difference in the precision between the two approaches?

Solution

The monotone framework is given by the lattice \( L = \mathbf{Var}_* \rightarrow \mathcal{P}(\{-+,0\}) \) and the following instantiations:

- \( l_1 \sqsubseteq l_2 \iff \forall x \in \mathbf{Var}_* : l_1(x) \subseteq l_2(x) \)
- \( \bot \in \mathbf{Var}_* \rightarrow \mathcal{P}(\{-+,0\}) \), \( \bot(x) = \emptyset \ \forall x \in \mathbf{Var}_* \)
- \( \top \in \mathbf{Var}_* \rightarrow \mathcal{P}(\{-+,0\}) \), \( \top(x) = \{-+,0\} \ \forall x \in \mathbf{Var}_* \)
- \( l_1 \sqcup l_2 \in \mathbf{Var}_* \rightarrow \mathcal{P}(\{-+,0\}) \), \( (l_1 \sqcup l_2)(x) = l_1(x) \cup l_2(x) \)
\[ F = \{ f : L \rightarrow L \mid f \text{ monotone} \} \]

\[ F = \text{flow}(S_\star) \]

\[ E = \{ \text{init}(S_\star) \} \]

\[ \nu = \bot \]

Let \( A_{VA} : \text{AExp} \rightarrow \widehat{\text{State}_{VA}} \rightarrow \mathcal{P}(-, 0, +) \) be the function that calculates the sign of an expression using the information in \( \widehat{\sigma} \). Then, \( f^{AV} \) is defined by:

\[
\begin{align*}
[x := a] : & \quad f^{AV}_x(\widehat{\sigma}) = \widehat{\sigma}[x \rightarrow A_{VA}[a] \widehat{\sigma}] \\
[\text{skip}] : & \quad f^{AV}_x(\widehat{\sigma}) = \widehat{\sigma} \\
[b] : & \quad f^{AV}_x(\widehat{\sigma}) = \widehat{\sigma}
\end{align*}
\]

There is no difference in precision for these approaches.