Prof. P. Thiemann, M. Geffken Summer Term 2014

Static Program Analysis

http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/

Exercise Sheet 3

15.05.2014

Exercise 1 (Posets)

1. Show that for the two partialy ordered sets (posets) $(\mathcal{P}(M), \subseteq)$ and $(\mathcal{P}(N), \subseteq)$ the product of the two posets is a poset

$$(\mathcal{P}(M) \times \mathcal{P}(N), \sqsubseteq).$$

The partial order $\sqsubseteq$ is defined as

$$(m_1, n_1) \sqsubseteq (m_2, n_2) \Leftrightarrow m_1 \subseteq m_2 \wedge n_1 \subseteq n_2.$$

You can assume that M and N are disjoint.

2. a) Let $(P_1, \sqsubseteq_1), \ldots, (P_n, \sqsubseteq_n)$ be posets. Show that the cartesian product $P_1 \times \cdots \times P_n$ and the relation $\sqsubseteq^n$, where $\sqsubseteq^n$ is defined as

$$(x_1, \ldots, x_n) \sqsubset^n (y_1, \ldots, y_n) \stackrel{def}{=} \exists i \in [1, n] : \forall j < i : x_j = y_j \wedge x_i \sqsubset_i y_i$$

$$(x_1, \ldots, x_n) \sqsubseteq^n (y_1, \ldots, y_n) \stackrel{def}{=} (x_1, \ldots, x_n) \sqsubset^n (y_1, \ldots, y_n) \vee \bigwedge_{i=1}^{n} x_i = y_i,$$

is a poset.

   b) Show that $(P_1 \times \cdots \times P_n, \sqsubseteq^n)$ is totally ordered if $(P_1, \sqsubseteq_1), \ldots, (P_n, \sqsubseteq_n)$ are totally ordered.

   c) What is the (unique) top/bottom element $\top/\bot$ of $(P_1 \times \cdots \times P_n, \sqsubseteq^n)$?

   d) What requirement(s) on $(P_1, \sqsubseteq_1), \ldots, (P_n, \sqsubseteq_n)$ need to be satisfied for $\top/\bot$ to exist in $(P_1 \times \cdots \times P_n, \sqsubseteq^n)$?

Submission In PDF format via email to `geffken AT informatik.uni-freiburg.de`. Please name your single file with the scheme: `ex03-name.pdf`, respectively.

- Deadline: **22.05.2014, 12:00**
- Late submissions will not be marked.
- Do not forget to write your name on the exercise sheet.