Exercise sheet 5

Exercise 1

The solution of Exercise 1 can be found at “The Calculus of Computation” by Bradley and Manna on pp. 140-142.

Exercise 2

See next pages.

@F
S_1 ;
...
S_n ;
@G

$F \rightarrow wp(G,\ S_1 ; \cdots ; S_n)$

1. $x > 0 \rightarrow wp(x \geq 0,\ x := x - k ;\ \text{assume } k \leq 1)$

$wp(x \geq 0,\ x := x - k ;\ \text{assume } k \leq 1) \equiv$

$\equiv wp(k \leq 1 \rightarrow x \geq 0,\ x := x - k) \equiv$

$\equiv k \leq 1 \rightarrow x - k \geq 0$

$x > 0 \rightarrow (k \leq 1 \rightarrow x \geq k)$ - not valid

$x = 0.5$

$k = 0.7$

2. $\top \longrightarrow wp(x \geq 0,\ \text{assume } k \leq x;\ x := x - k)$

$wp(x \geq 0,\ \text{assume } k \leq x;\ x := x - k) =$

$= wp(x - k \geq 0,\ \text{assume } k \leq x) =$

$= k \leq x \longrightarrow x \geq k$ — valid

3. $T \longrightarrow wp\ (x \geq 0,\ x := x-k;\ \text{assume } k \leq x)$

$wp\ (x \geq 0,\ x := x-k;\ \text{assume } k \leq x) \equiv$

$\equiv wp\ (k \leq x \rightarrow x \geq 0,\ x := x-k) \equiv$

$\equiv \quad k \leq x-k \rightarrow x-k \geq 0 \equiv$

$\equiv \quad x \geq 2k \rightarrow x \geq k$

$T \rightarrow F \equiv F \equiv x \geq 2k \rightarrow x \geq k$ – not valid

$k = -1$

$x = -1.5$

4. $k \geq 0 \longrightarrow wp\left(x \geq 0,\ x := x - k;\ \text{assume } k \leq x\right)$

$wp\left(x \geq 0,\ x := x - k;\ \text{assume } k \leq x\right) =$

$\geq\ wp\left(k \leq x \rightarrow x \geq 0,\ x := x - k\right) =$

$\geq\ k \leq x - k \longrightarrow x - k \geq 0 =$

$=\ x \geq 2k \rightarrow x \geq k$

$k \geq 0 \longrightarrow \left(x \geq 2k \rightarrow x \geq k\right)$ - valid

5. $y \geq 0 \rightarrow wp(x+2y \geq 3,\ x := x+1;\ \text{assume } x>0;\ y := y+x)$

$wp(\dots) \equiv wp(x+2y+2x \geq 3,\ x := x+1;\ \text{assume } x>0) \equiv$

$\equiv wp(3x+2y \geq 3,\ x := x+1;\ \text{assume } x>0) \equiv$

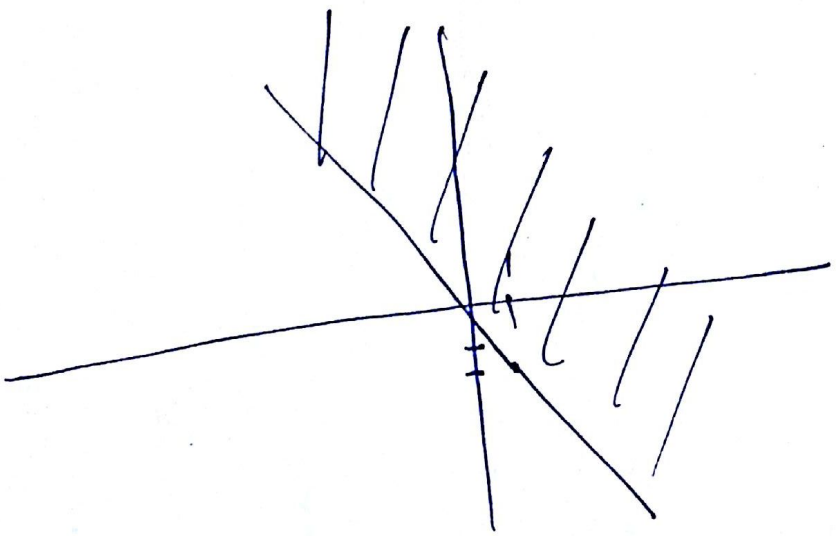
$\equiv wp(x>0 \rightarrow 3x+2y \geq 3,\ x := x+1) \equiv$

$\equiv x+1>0 \rightarrow 3x+3+2y \geq 3 \equiv$

$\equiv x+1>0 \rightarrow 3x+2y \geq 0$

$y \geq 0 \rightarrow (x+1>0 \rightarrow 3x+2y \geq 0)$

$y \geq -\frac{3}{2}x$



$y=0$

$x=-0.5$ - not valid