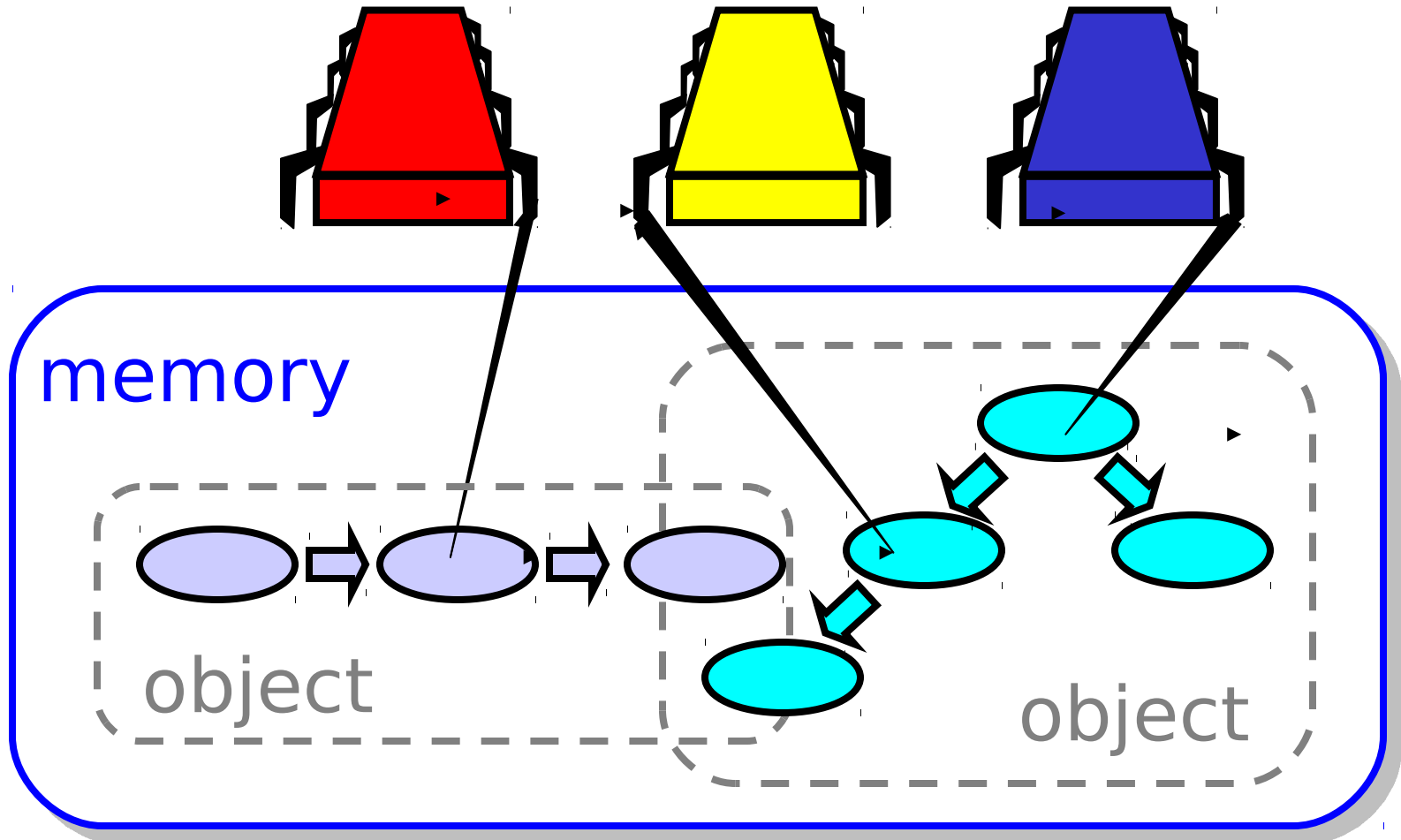


Concurrent Objects

Companion slides for
The Art of Multiprocessor
Programming
by Maurice Herlihy & Nir Shavit

Concurrent Computaton



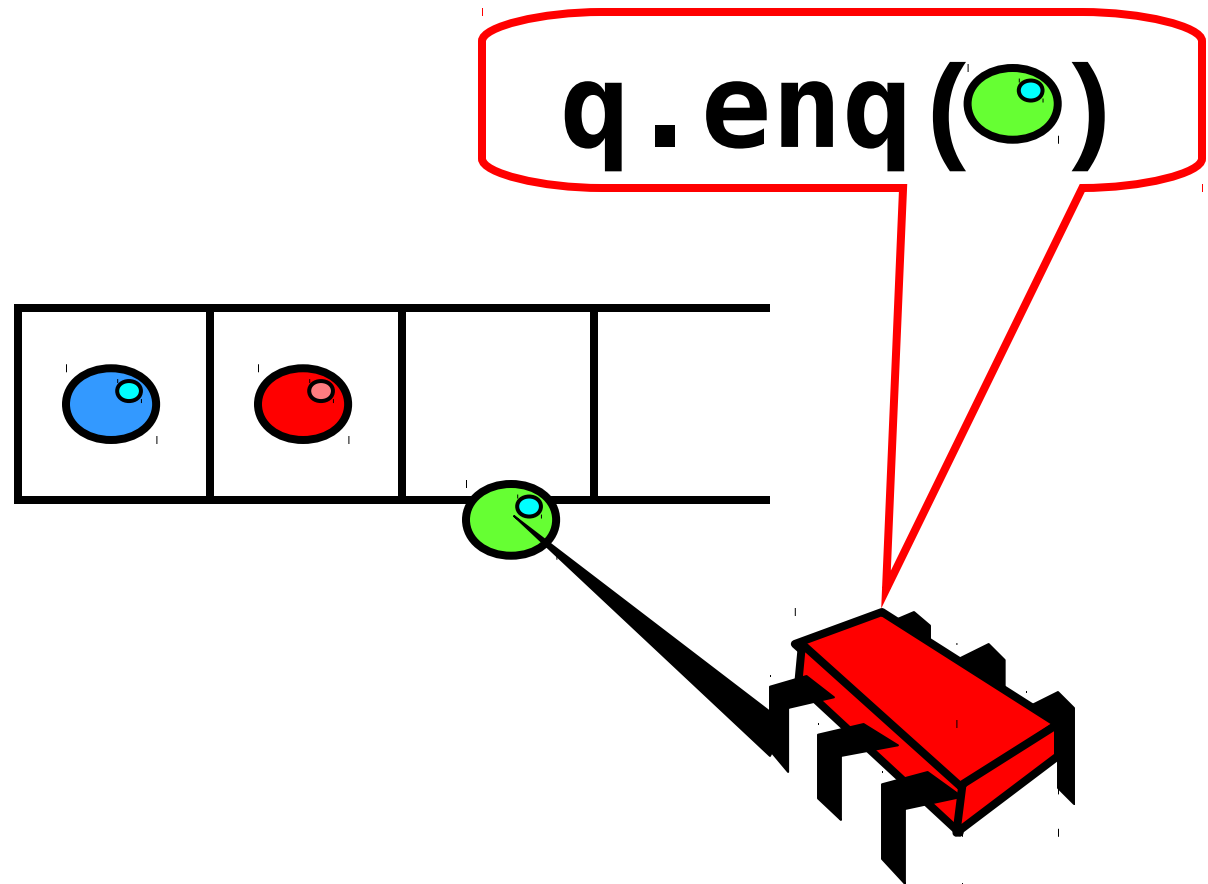
Objectivism

- What is a concurrent object?
 - How do we **describe** one?
 - How do we **implement** one?
 - How do we **tell if we're right**?

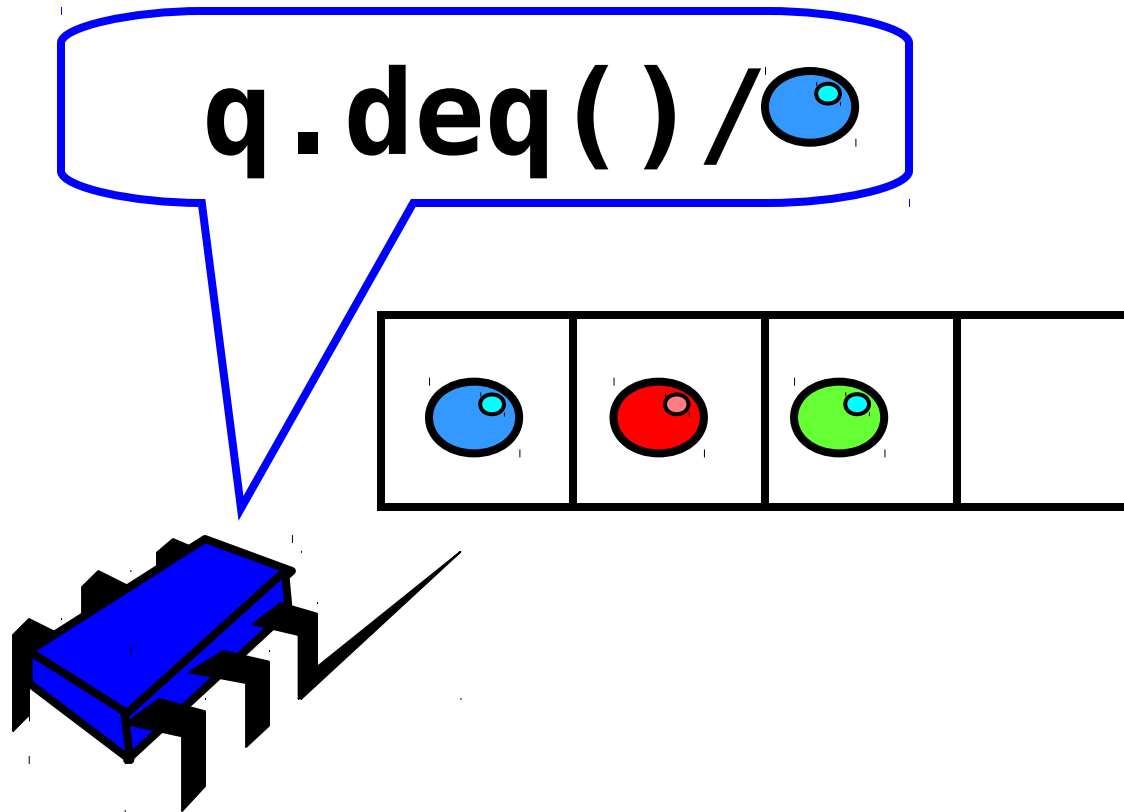
Objectivism

- What is a concurrent object?
 - How do we **describe** one?
 - How do we **tell if we're right**?

FIFO Queue: Enqueue Method



FIFO Queue: Dequeue Method

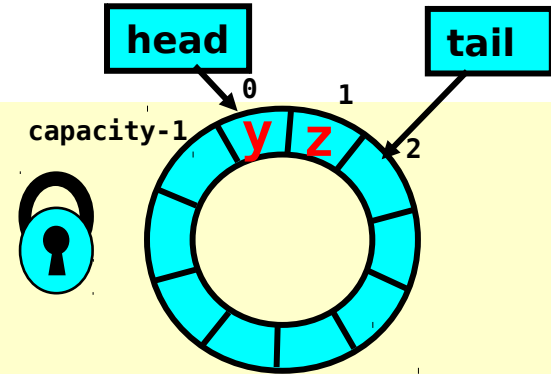


A Lock-Based Queue

```
class LockBasedQueue<T> {  
    int head, tail;  
    T[] items;  
    Lock lock;  
    public LockBasedQueue(int capacity) {  
        head = 0; tail = 0;  
        lock = new ReentrantLock();  
        items = (T[]) new Object[capacity];  
    }  
}
```

A Lock-Based Queue

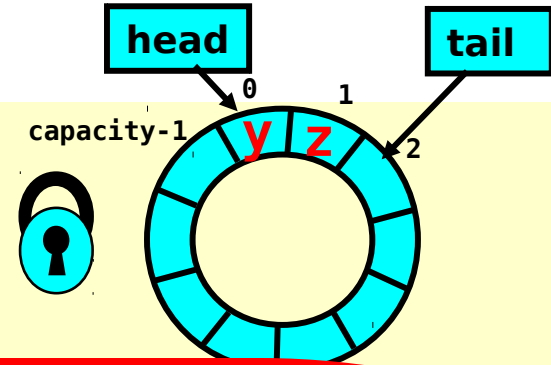
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    }  
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```



Queue fields
protected by single
shared lock

A Lock-Based Queue

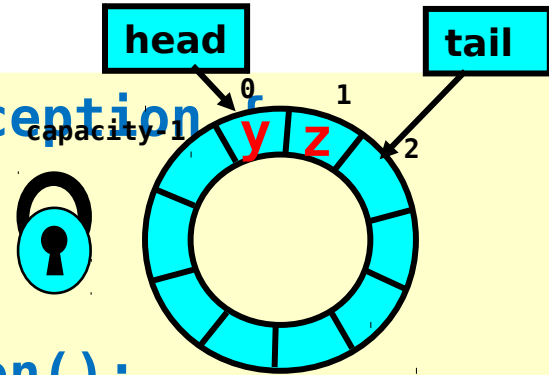
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    }  
}
```



Initially head =
tail

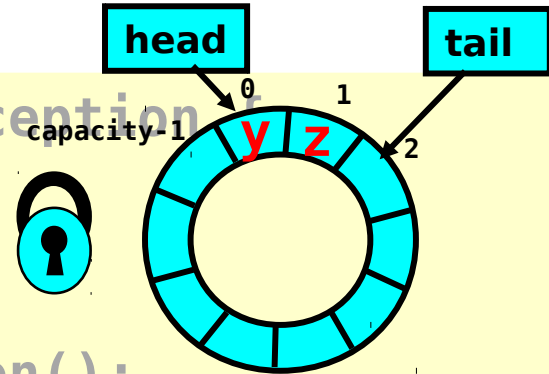
Implementation: Deq

```
public T deq() throws EmptyException {
    lock.lock();
    try {
        if (tail == head)
            throw new EmptyException();
        T x = items[head % items.length];
        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```



Implementation: Deq

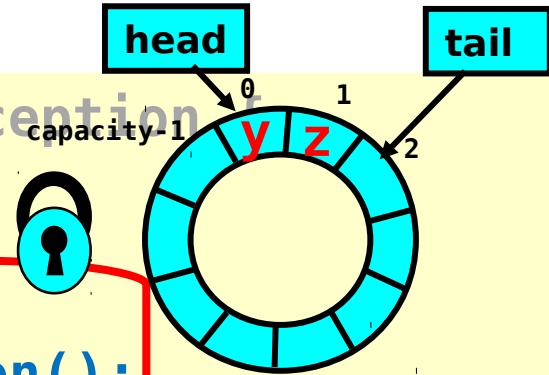
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        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```



Method calls
mutually exclusive

Implementation: Deq

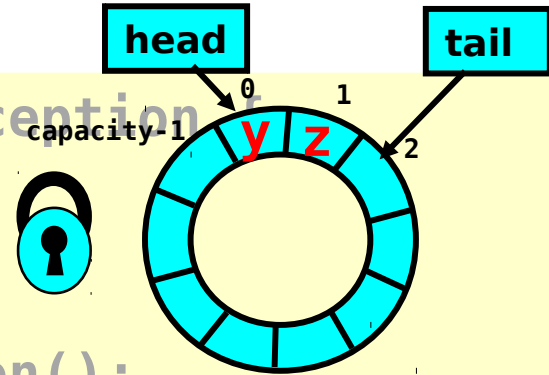
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}
```



If queue empty
throw exception

Implementation: Deq

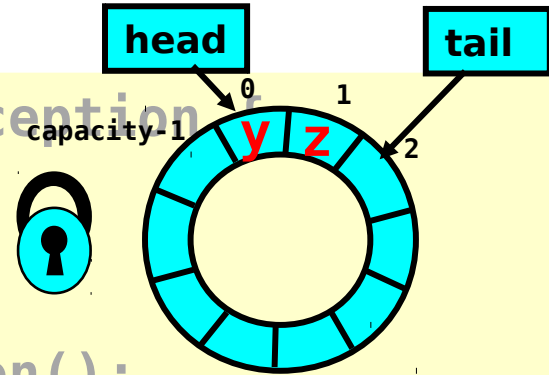
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        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```



Queue not empty:
remove item and
update
head

Implementation: Deq

```
public T deq() throws EmptyException {
    lock.lock();
    try {
        if (tail == head)
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        T x = items[head % items.length];
        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```

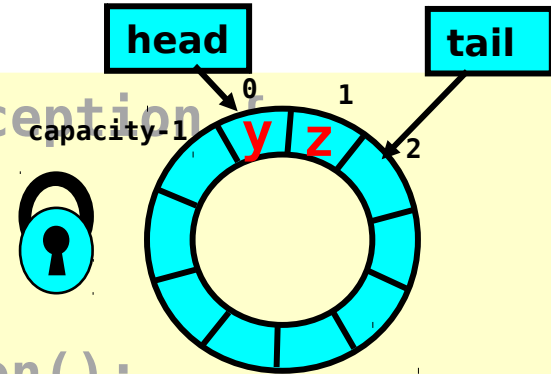


return x;

Return result

Implementation: Deq

```
public T deq() throws EmptyException {
    lock.lock();
    try {
        if (tail == head)
            throw new EmptyException();
        T x = items[head % items.length];
        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```



Release lock no matter what!

Implementation: Deq

```
public T deq() throws EmptyException {
    lock.lock();
    try {
        if (tail == head)
            throw new EmptyException();
        T x = items[head % items.length];
        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```

**Should be correct because
modifications are mutually
exclusive...**

Now consider the following implementation

- The same thing without mutual exclusion
- For simplicity, only two threads
 - One thread **enq only**
 - The other **deq only**

Wait-free 2-Thread Queue

```
public class WaitFreeQueue {  
  
    int head = 0, tail = 0;  
    items = (T[]) new Object[capacity];  
  
    public void enq(Item x) {  
        while (tail-head == capacity); // busy-wait  
        items[tail % capacity] = x; tail++;  
    }  
    public Item deq() {  
        while (tail == head); // busy-wait  
        Item item = items[head % capacity]; head++;  
        return item;  
    }  
}
```

Wait-free 2-Thread Queue

```
public class LockFreeQueue {
```

```
    int head = 0, tail = 0;
```

```
    items = (T[]) new Object[capacity]
```

```
    public void enq(Item x) {
```

```
        while (tail-head == capacity); // busy-wait
```

```
        items[tail % capacity] = x; tail++;
```

```
    }
```

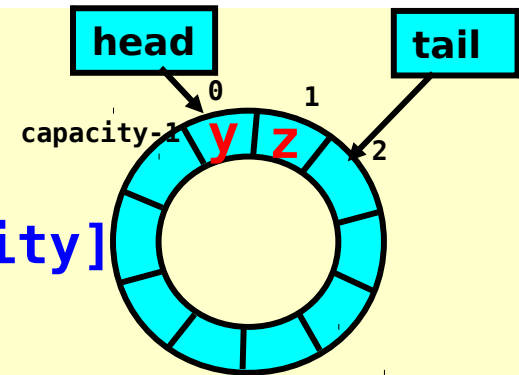
```
    public Item deq() {
```

```
        while (tail == head); // busy-wait
```

```
        Item item = items[head % capacity]; head++;
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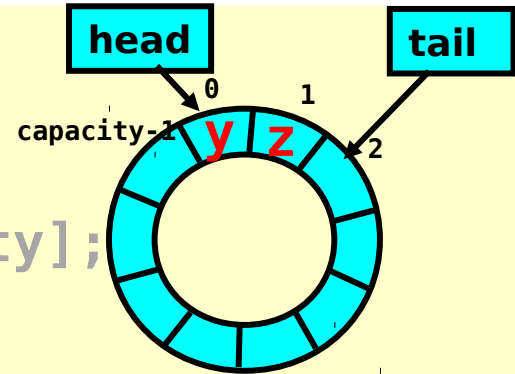
```
        return item;
```

```
    }}
```



Lock-free 2-Thread Queue

```
public class LockFreeQueue {  
    int head = 0, tail = 0;  
    items = (T[])new Object[capacity];  
  
    public void enq(Item x) {  
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    }  
    public Item deq() {  
        while (tail == head);  
        Item item = items[head];  
        return item;  
    }  
}
```



`items[tail % capacity] = x; tail++;`

How do we define "correct" when modifications are not mutually exclusive?

Queue is up

LOCK!

Defining concurrent queue implementations

- Need a way to specify a concurrent queue object
- Need a way to prove that an algorithm implements the object's specification
- Lets talk about object specifications ...

Correctness and Progress

- In a concurrent setting, we need to specify both the safety and the liveness properties of an object
- Need a way to define
 - when an implementation is correct
 - the conditions under which it guarantees progress

Lets begin with correctness

Sequential Objects

- Each object has a ***state***
 - Usually given by a set of ***fields***
 - Queue example: sequence of items
- Each object has a set of ***methods***
 - Only way to manipulate state
 - Queue example: **enq** and **deq** methods

Sequential Specifications

- If (precondition)
 - the object is in such-and-such a state
 - before you call the method,
- Then (postcondition)
 - the method will return a particular value
 - or throw a particular exception.
- and (postcondition, con't)
 - the object will be in some other state
 - when the method returns,

Pre- and Postconditions for Dequeue

- **Precondition:**
 - Queue is non-empty
- **Postcondition:**
 - Returns first item in queue
- **Postcondition:**
 - Removes first item in queue

Pre- and Postconditions for Dequeue

- **Precondition:**
 - Queue is empty
- **Postcondition:**
 - Throws Empty exception
- **Postcondition:**
 - Queue state unchanged

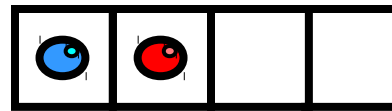
Why Sequential Specifications Totally Rock

- Interactions among methods captured by side-effects on object state
 - State meaningful between method calls
- Documentation size linear in number of methods
 - Each method described in isolation
- Can add new methods
 - Without changing descriptions of old methods

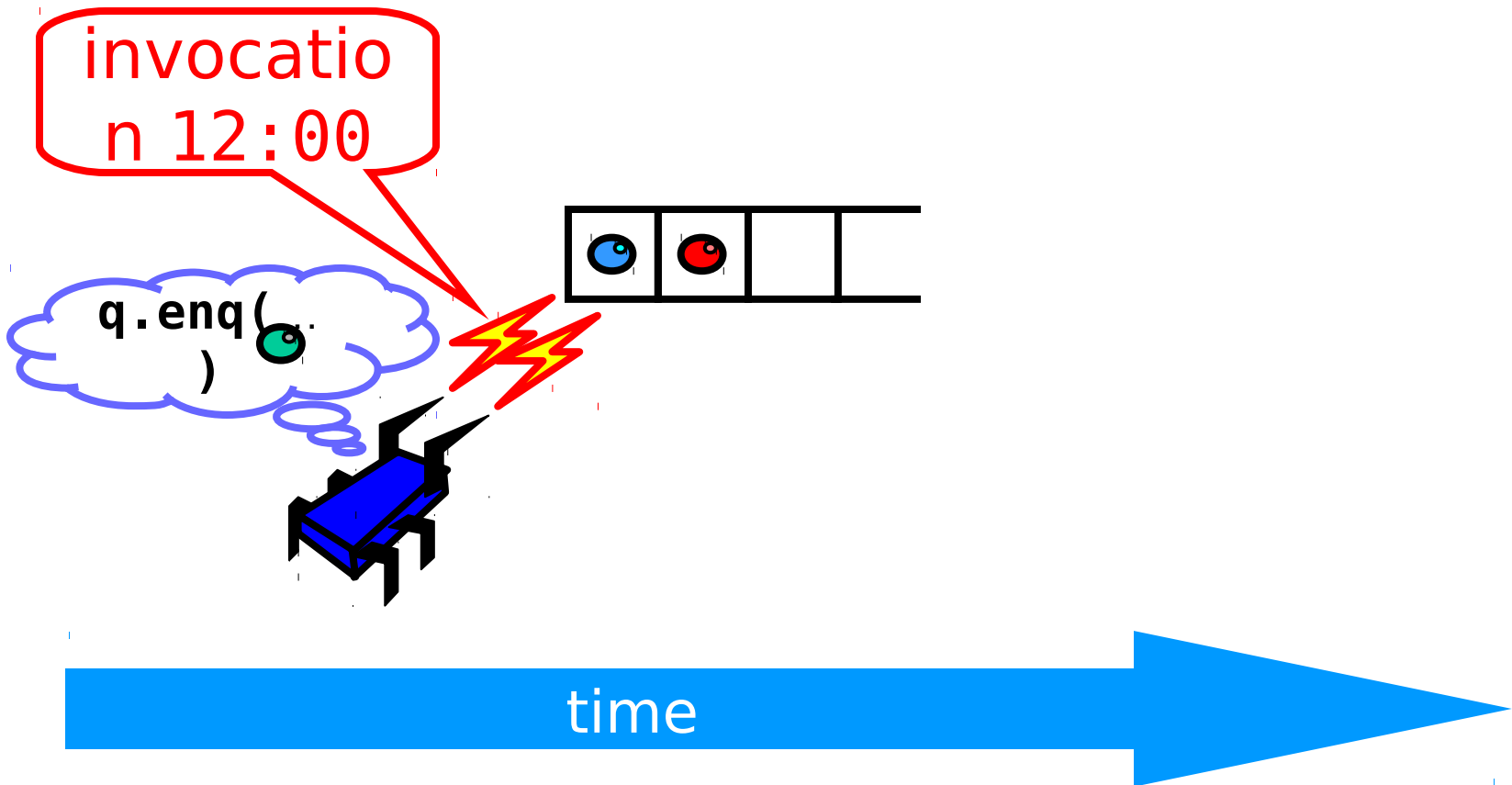
What About Concurrent Specifications? x

- Methods?
- Documentation?
- Adding new methods?

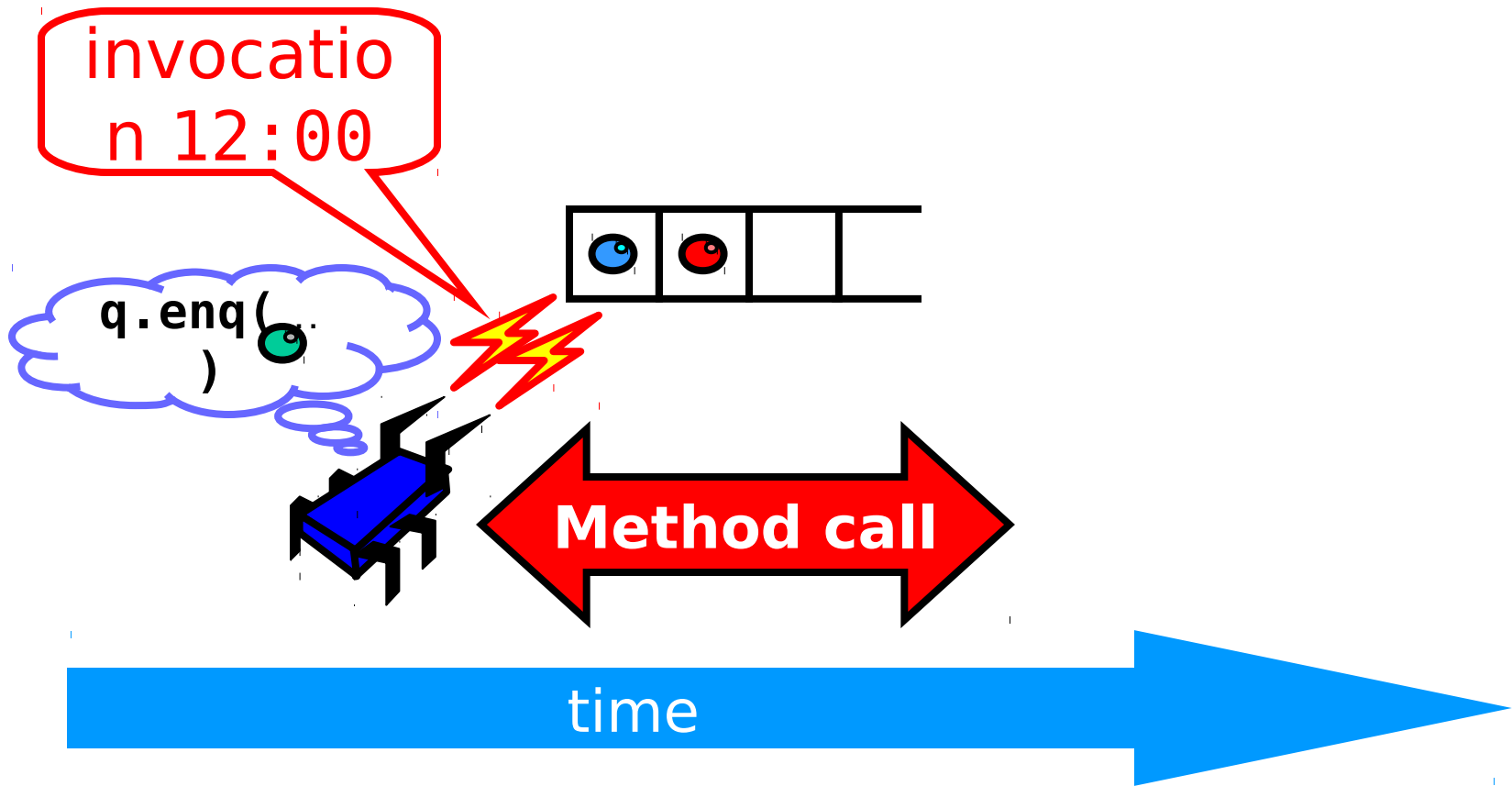
Methods Take Time



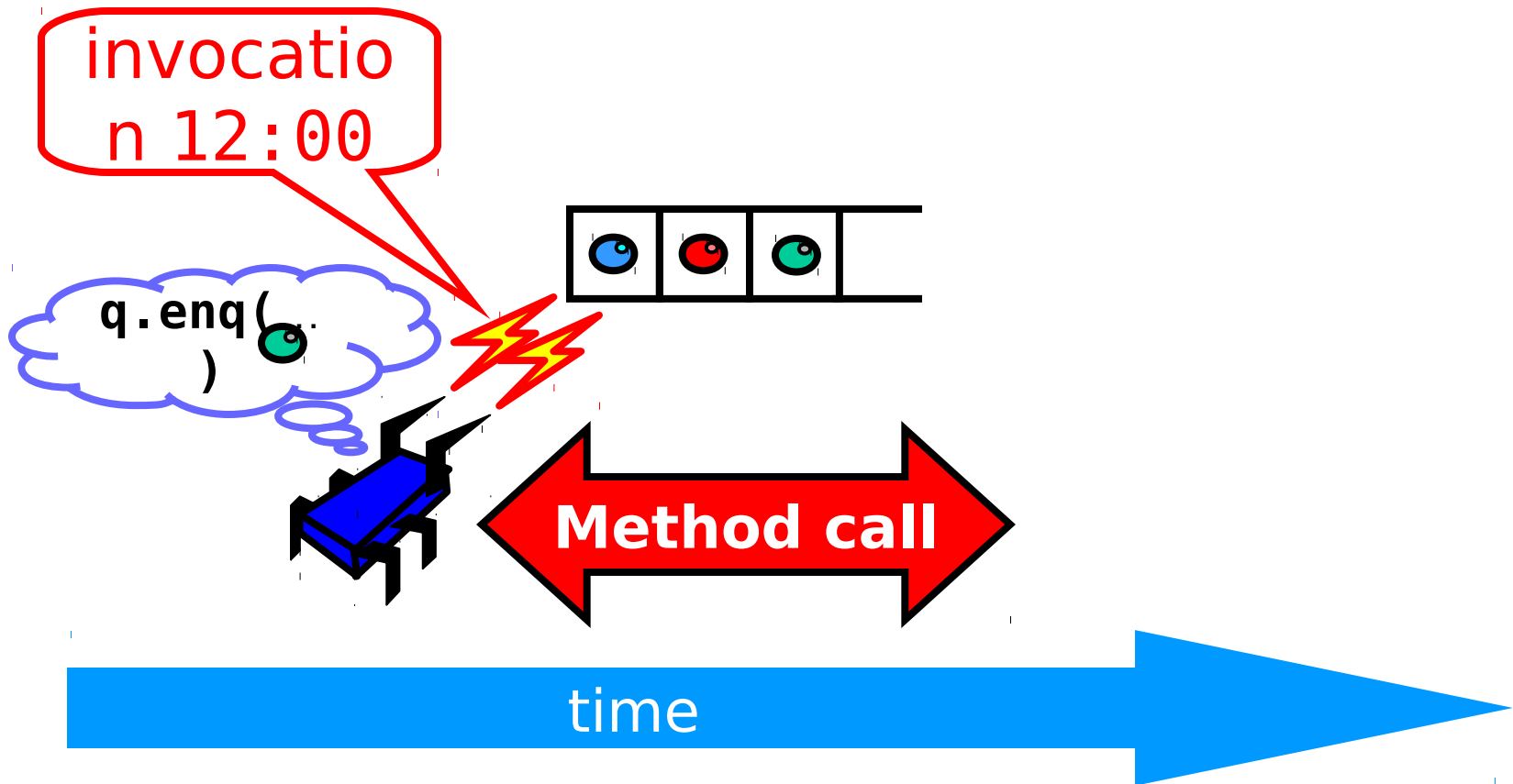
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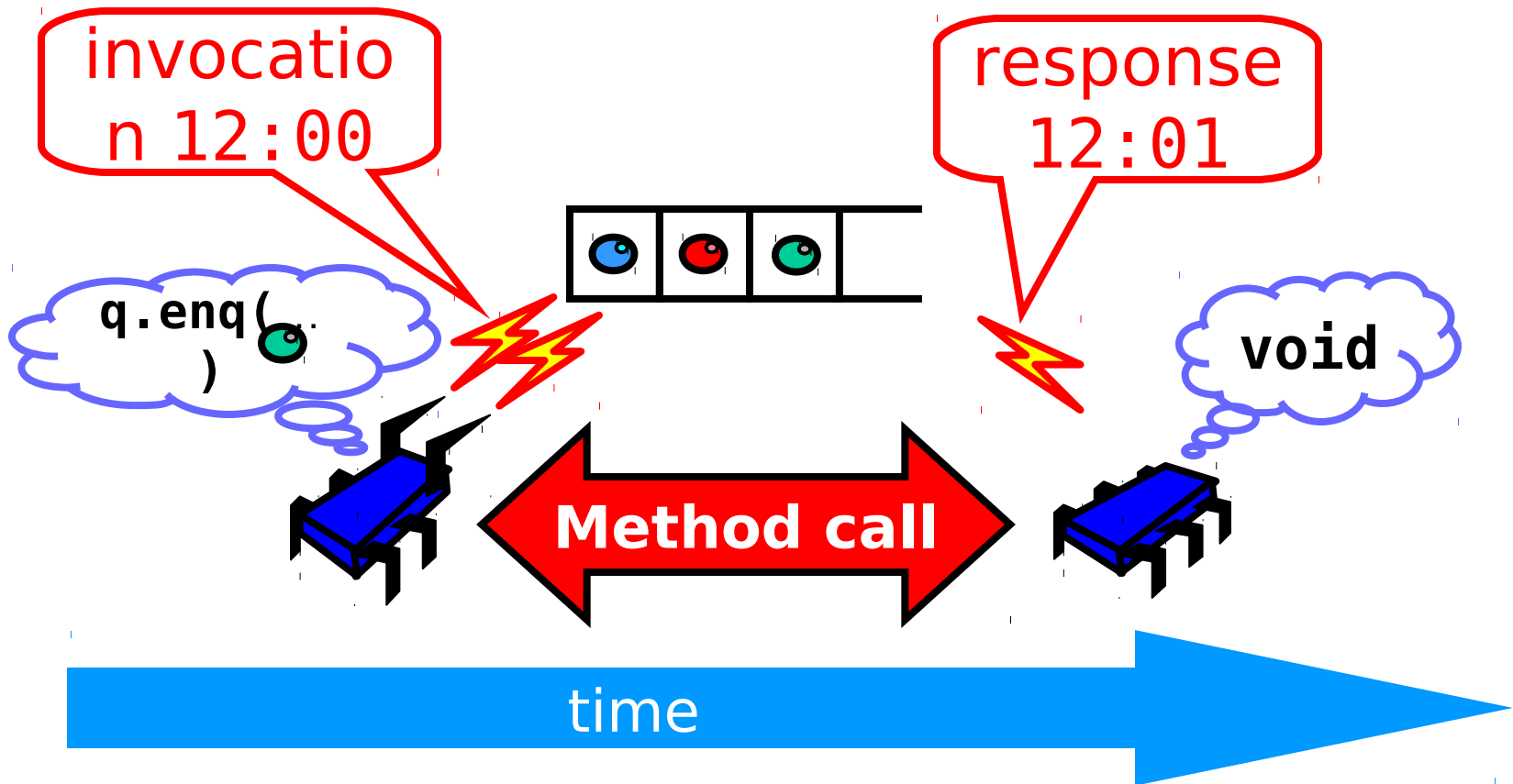
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Methods Take Time



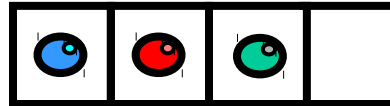
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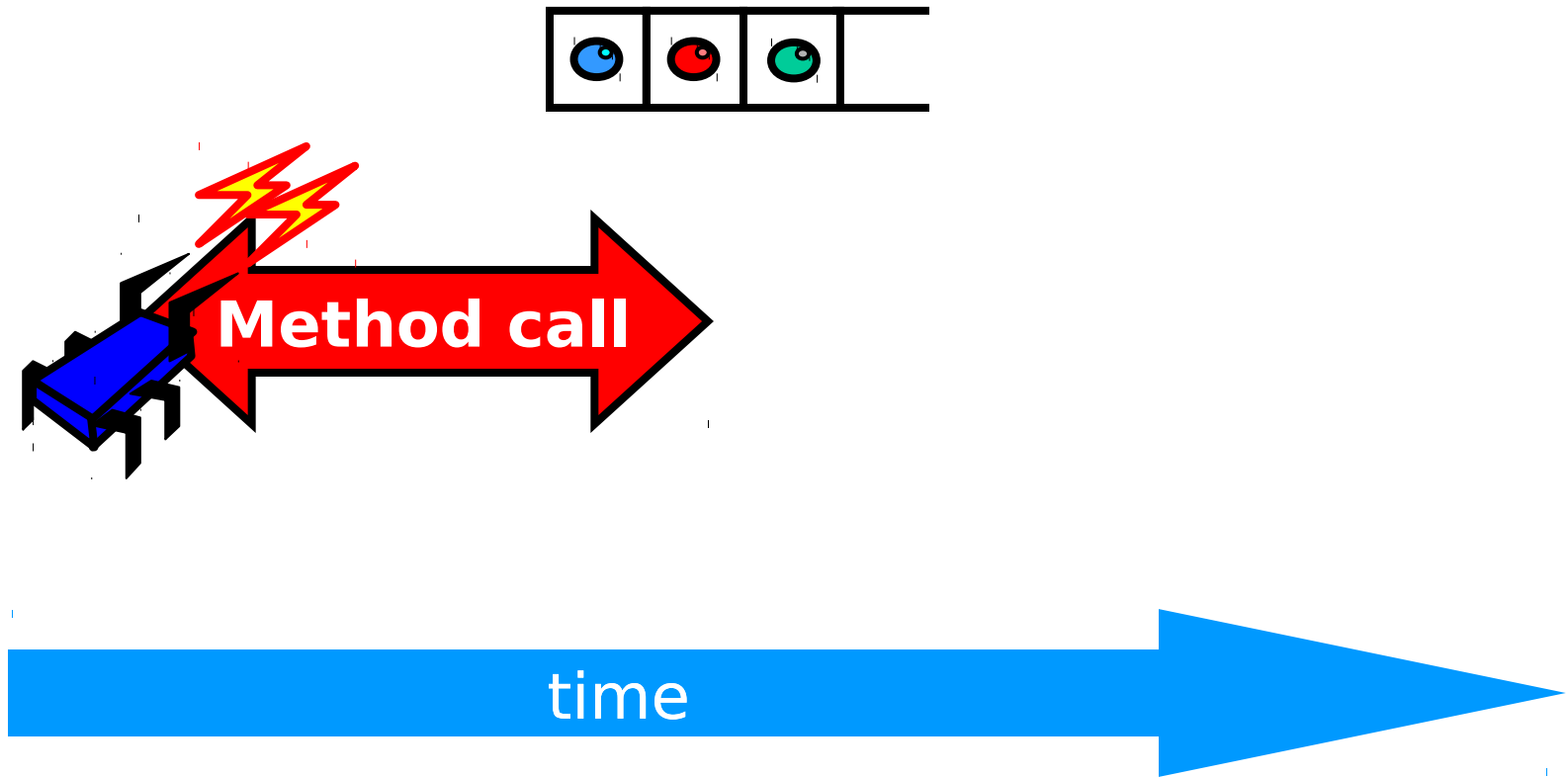
Sequential vs Concurrent

- Sequential
 - Methods take time? Who knew?
- Concurrent
 - Method call is not an event
 - Method call is an interval.

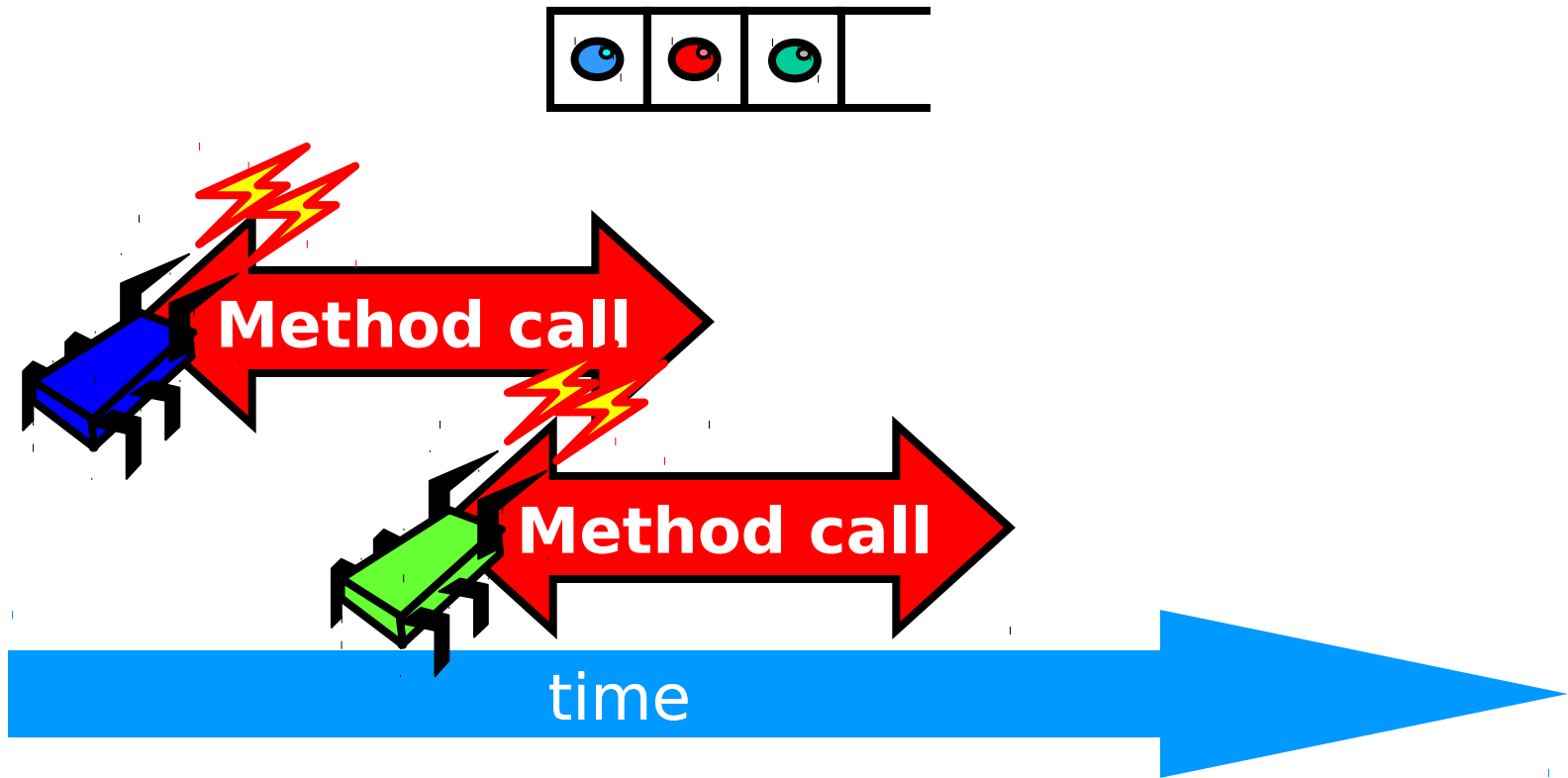
Concurrent Methods Take Overlapping Time



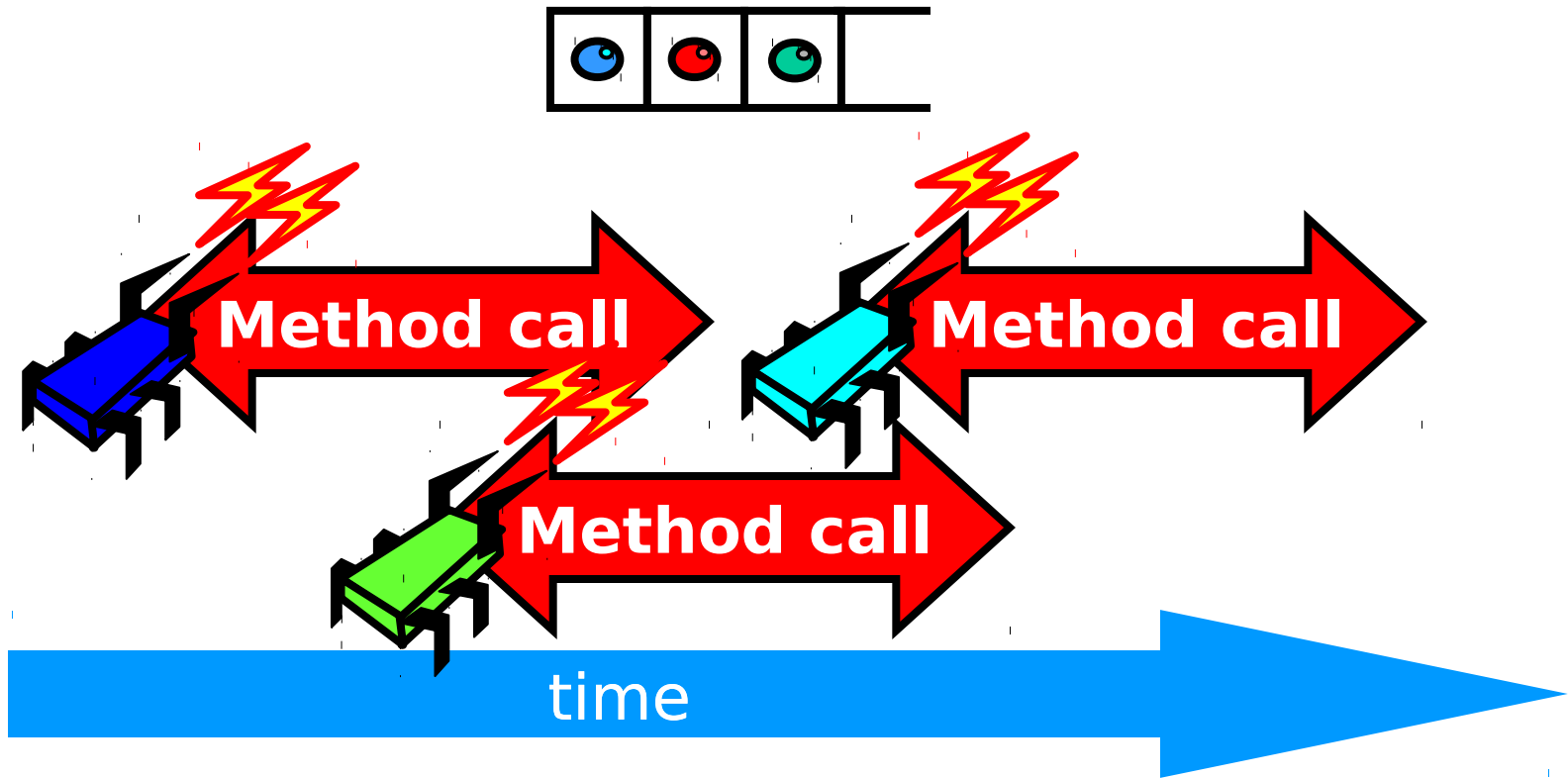
Concurrent Methods Take Overlapping Time



Concurrent Methods Take Overlapping Time



Concurrent Methods Take Overlapping Time



Sequential vs Concurrent

- Sequential:
 - Object needs meaningful state only between method calls
- Concurrent
 - Because method calls overlap, object might ***never*** be between method calls

Sequential vs Concurrent

- Sequential:
 - Each method described in isolation
- Concurrent
 - Must characterize **all** possible interactions with concurrent calls
 - What if two enqs overlap?
 - Two deqs? enq and deq? ...

Sequential vs Concurrent

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:
 - Everything can potentially interact with everything else

Sequential vs Concurrent

- Sequential:
 - Can add new methods without affecting older methods
- Concurrent:
 - Everything can potentially interact with everything else

Panic!

The Big Question

- What does it **mean** for a *concurrent* object to be correct?
 - What *is* a concurrent FIFO queue?
 - FIFO means strict temporal order
 - Concurrent means ambiguous temporal order

...Intuitively

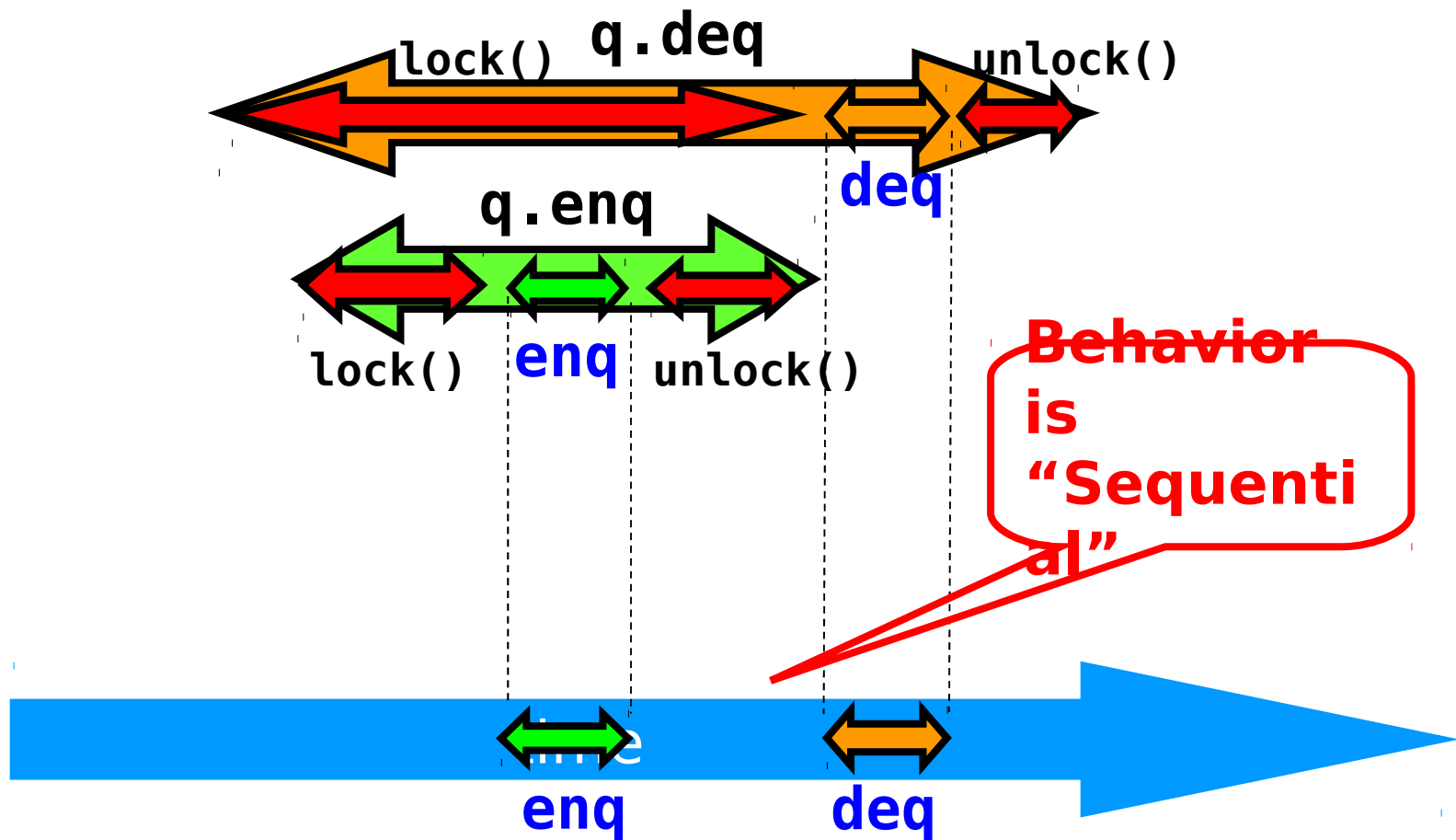
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        return x;
    } finally {
        lock.unlock();
    }
}
```

...Intuitively

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    lock.lock();  
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        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

All modifications
of queue are done
mutually exclusive

Lets capture the idea of describing the concurrent via the sequential



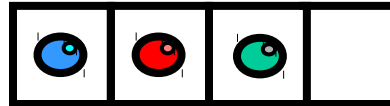
Linearizability

- Each method should
 - “take effect”
 - Instantaneously
 - Between invocation and response events
- Object is correct if this “sequential” behavior is correct
- Any such concurrent object is
 - **Linearizable**TM

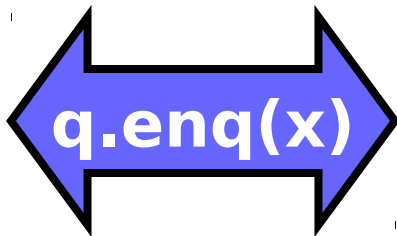
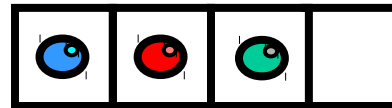
Is it really about the object? x

- Each method should
 - “take effect”
 - Instantaneously
 - Between invocation and response events
- Sounds like a property of an execution...
- A linearizable object: one all of whose possible executions are linearizable

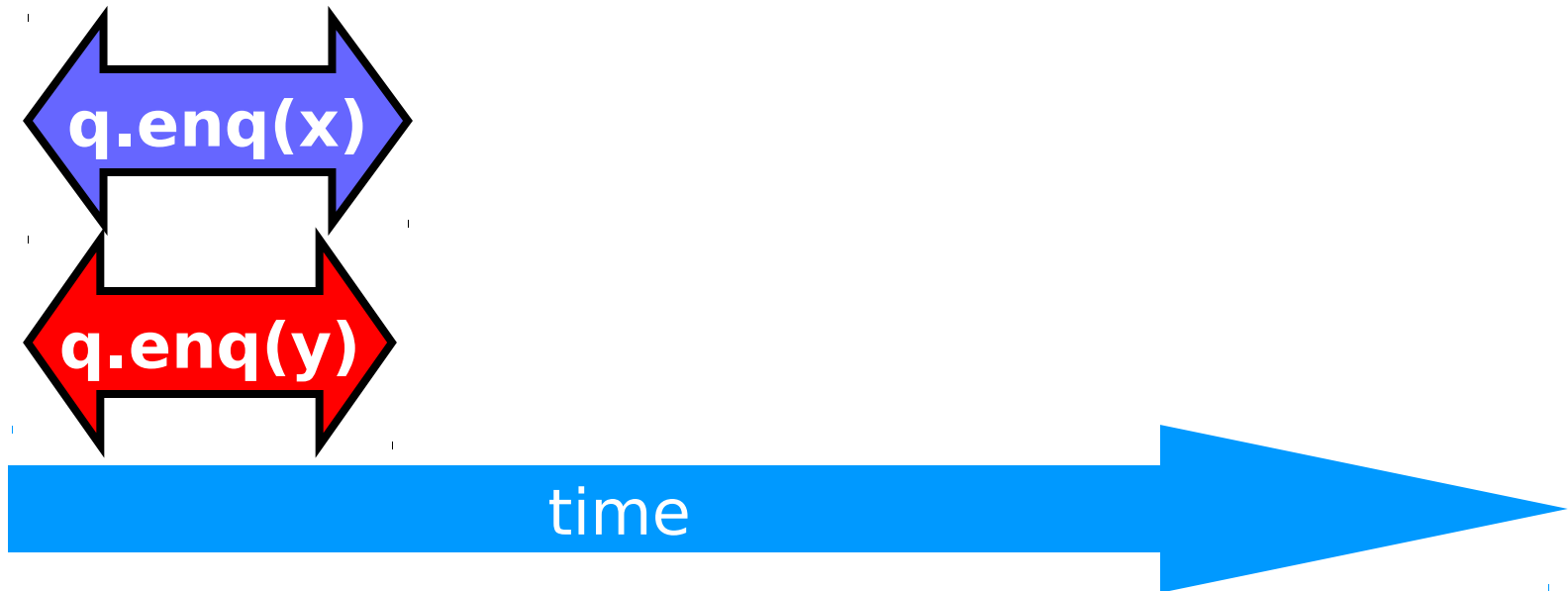
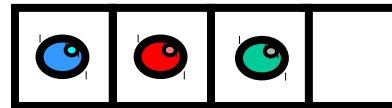
Example



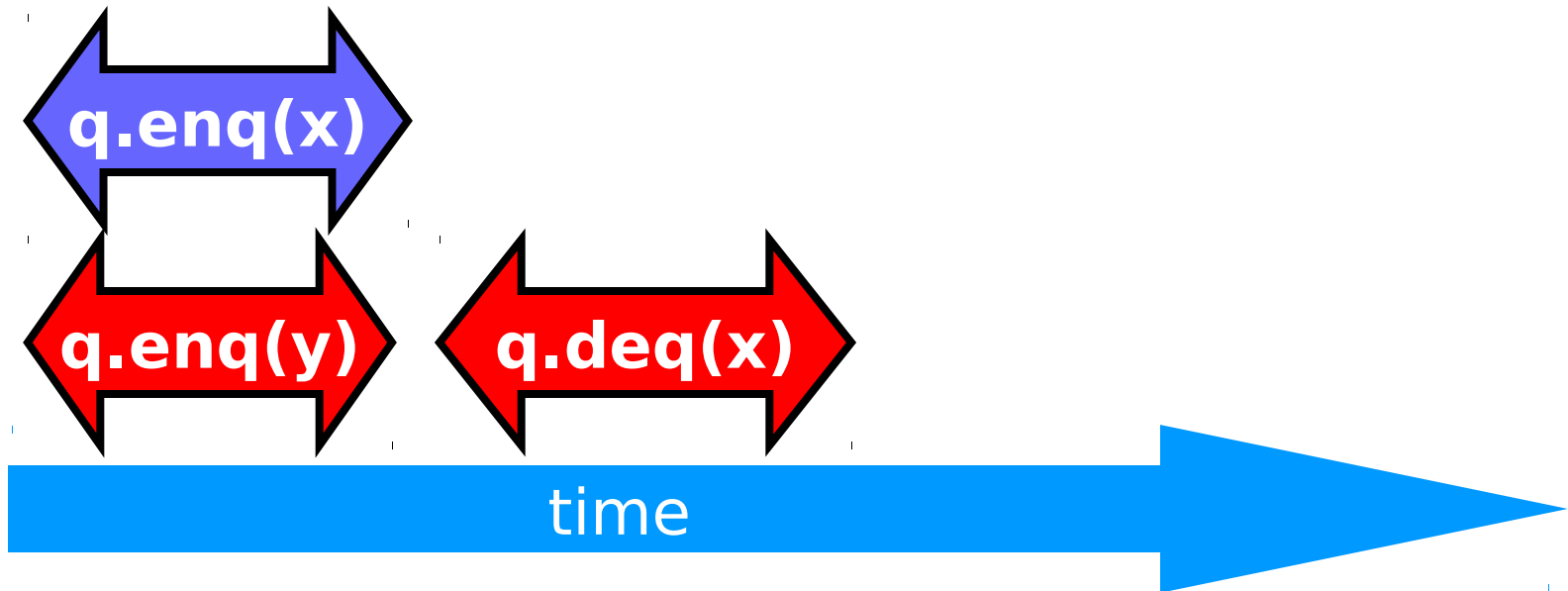
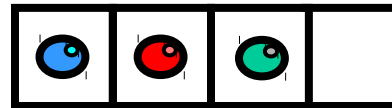
Example



Example

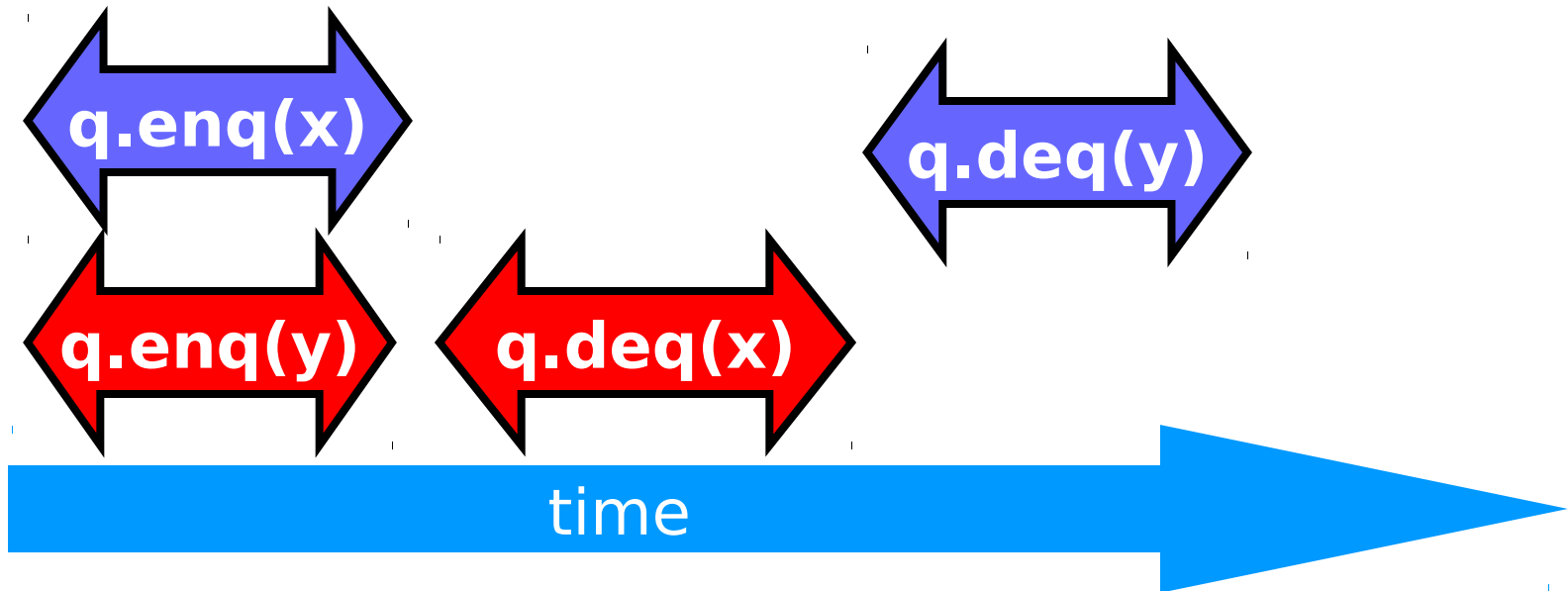
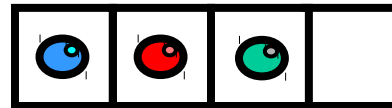


Example

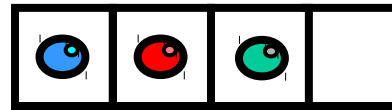




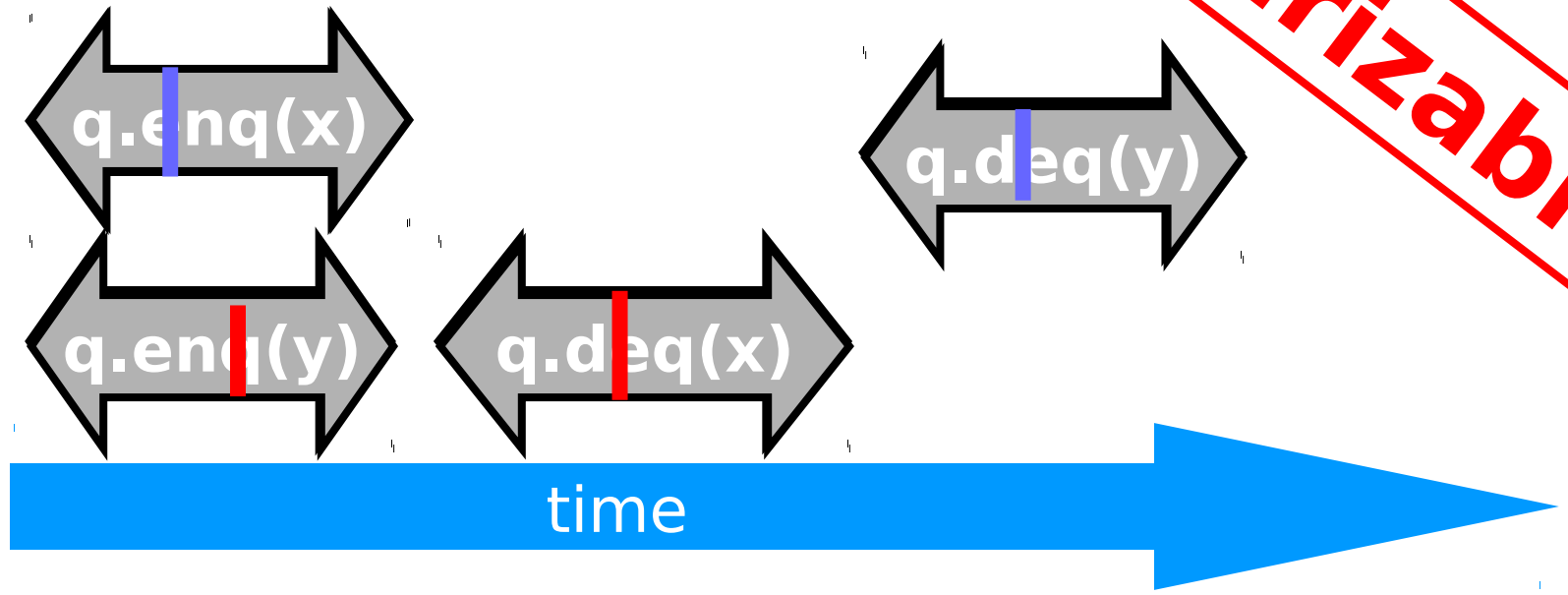
Example



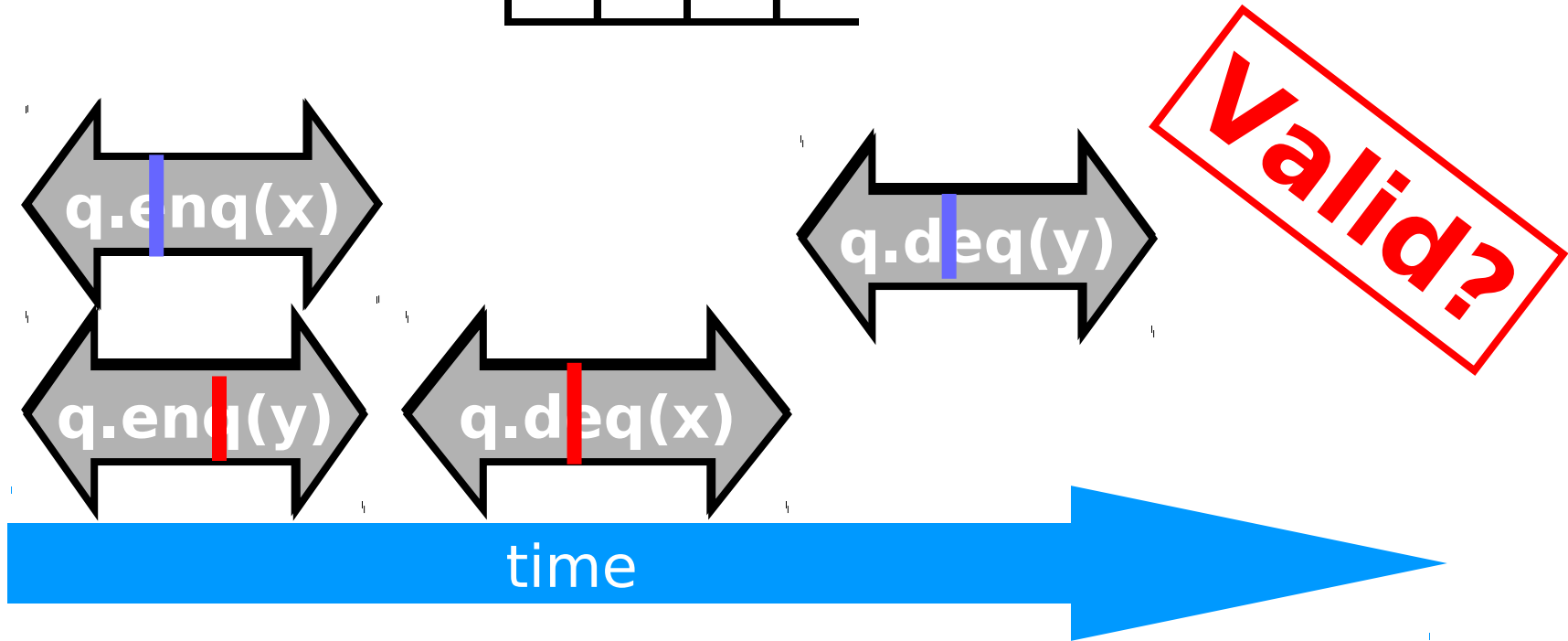
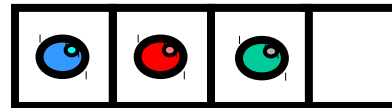
Example



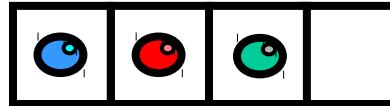
linearizable



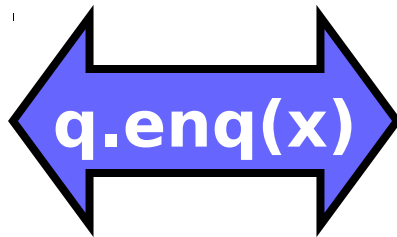
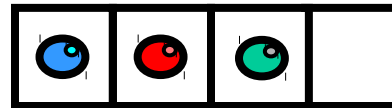
Example



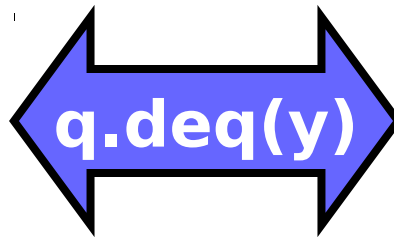
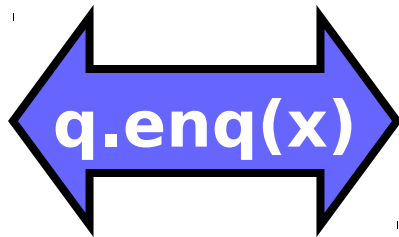
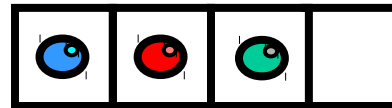
Example



Example

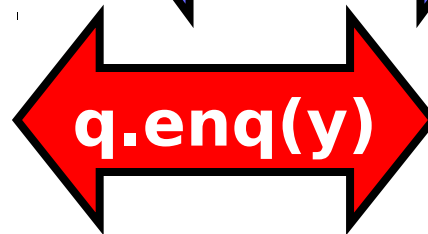
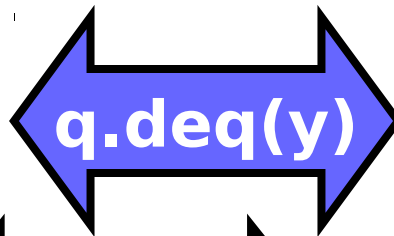
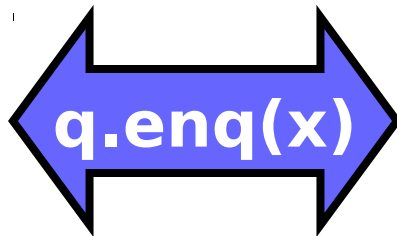
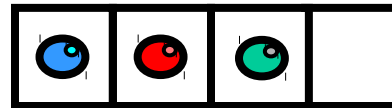


Example



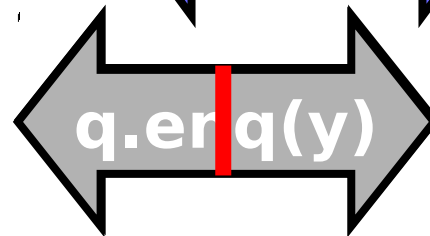
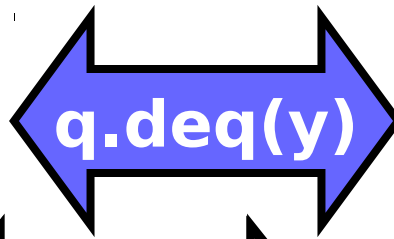
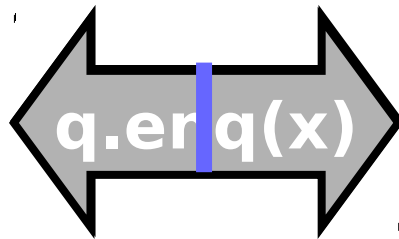
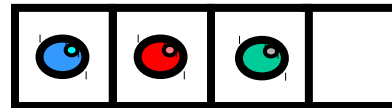


Example



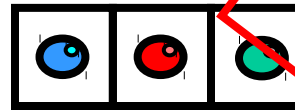


Example

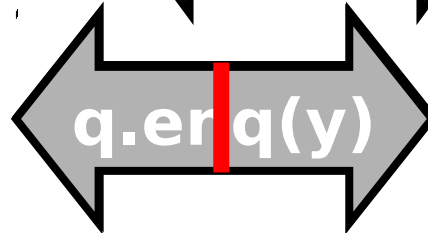
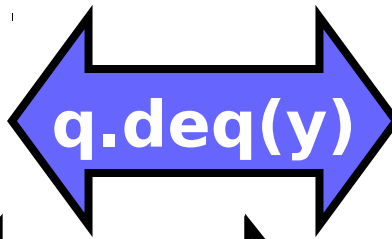
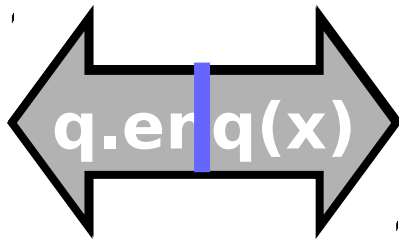




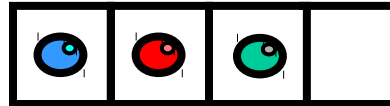
Example



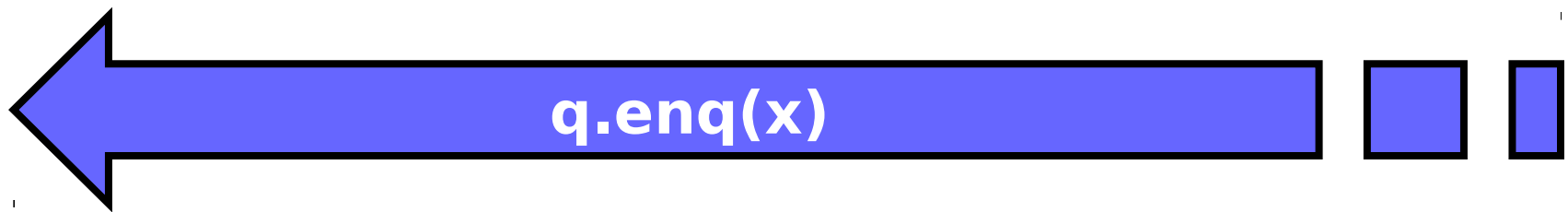
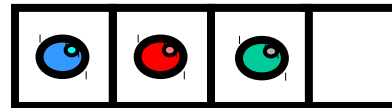
not linearizable



Example

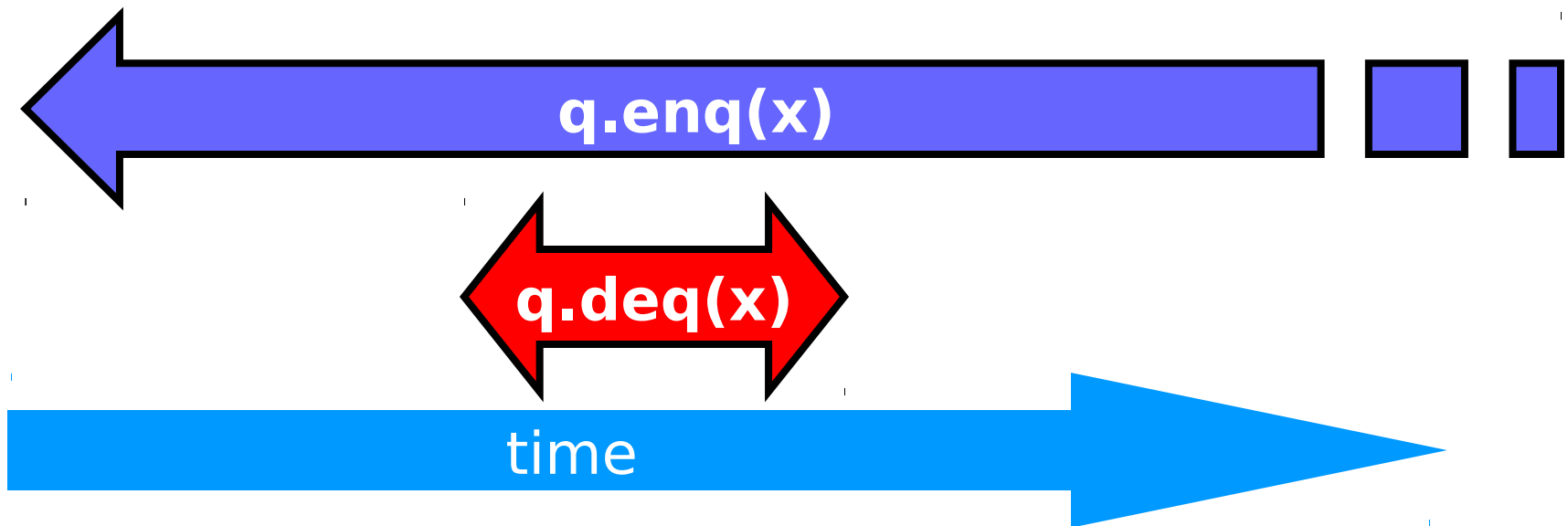
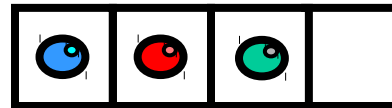


Example



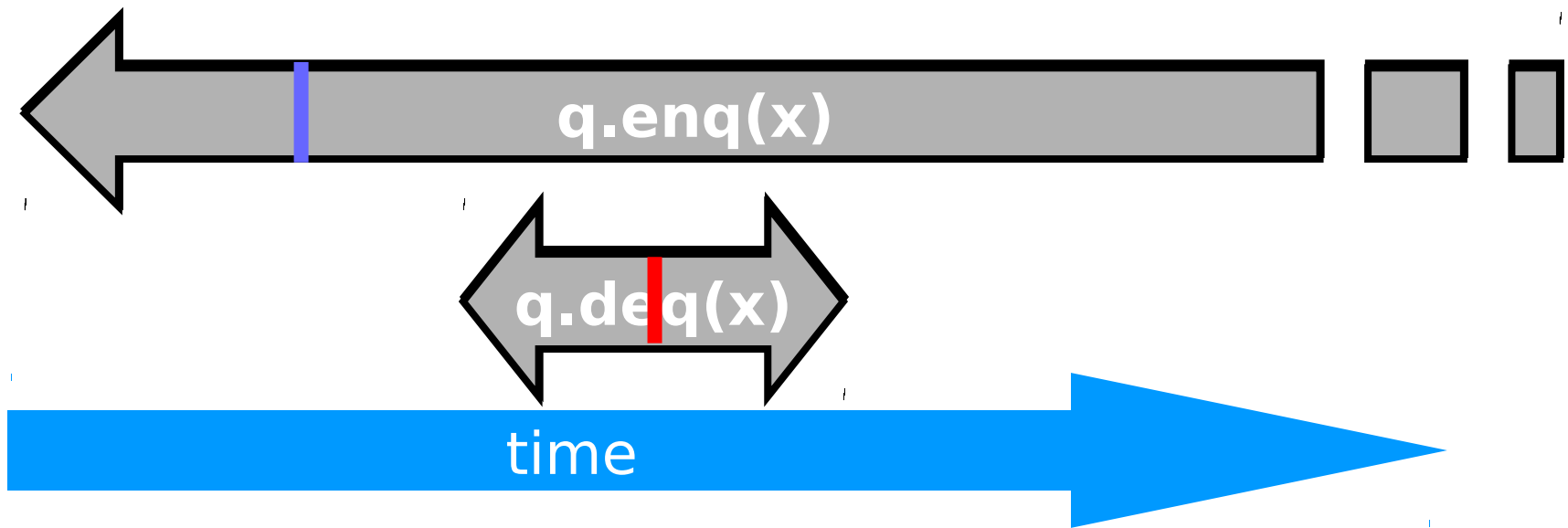
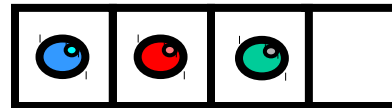


Example



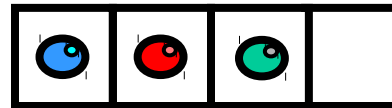


Example

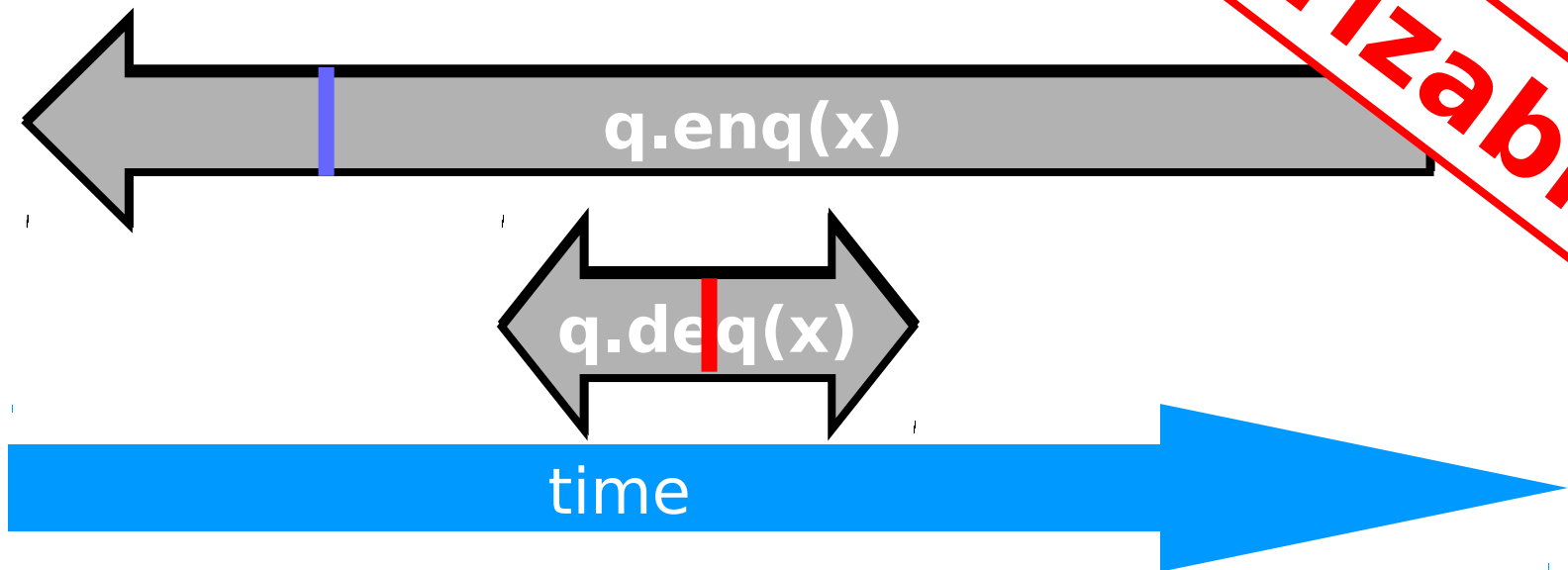




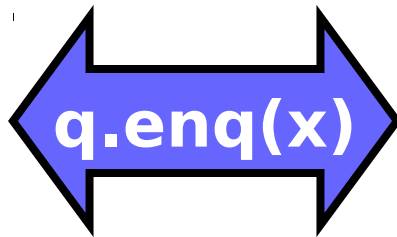
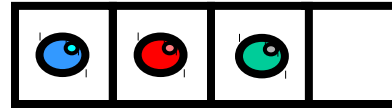
Example



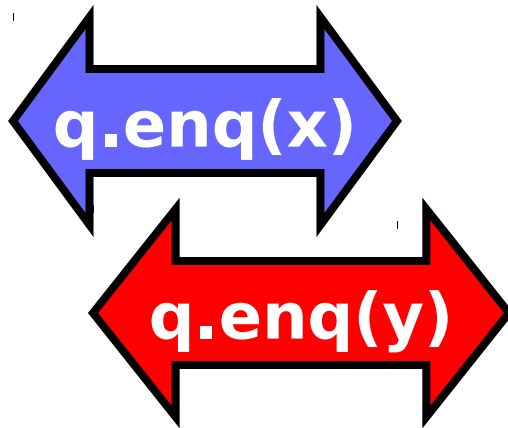
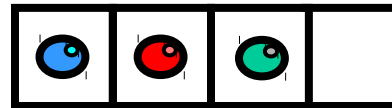
linearizable



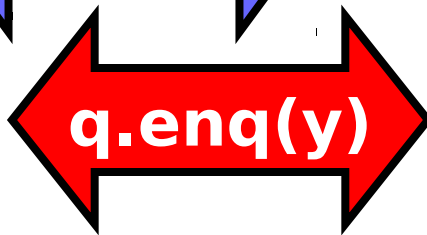
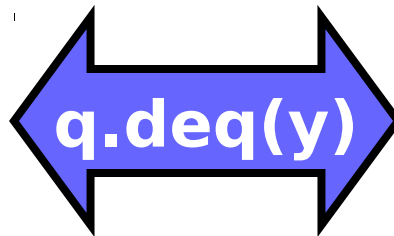
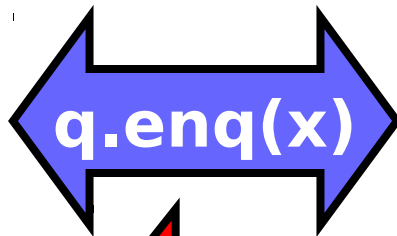
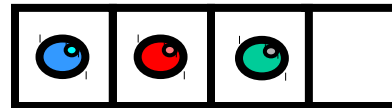
Example



Example

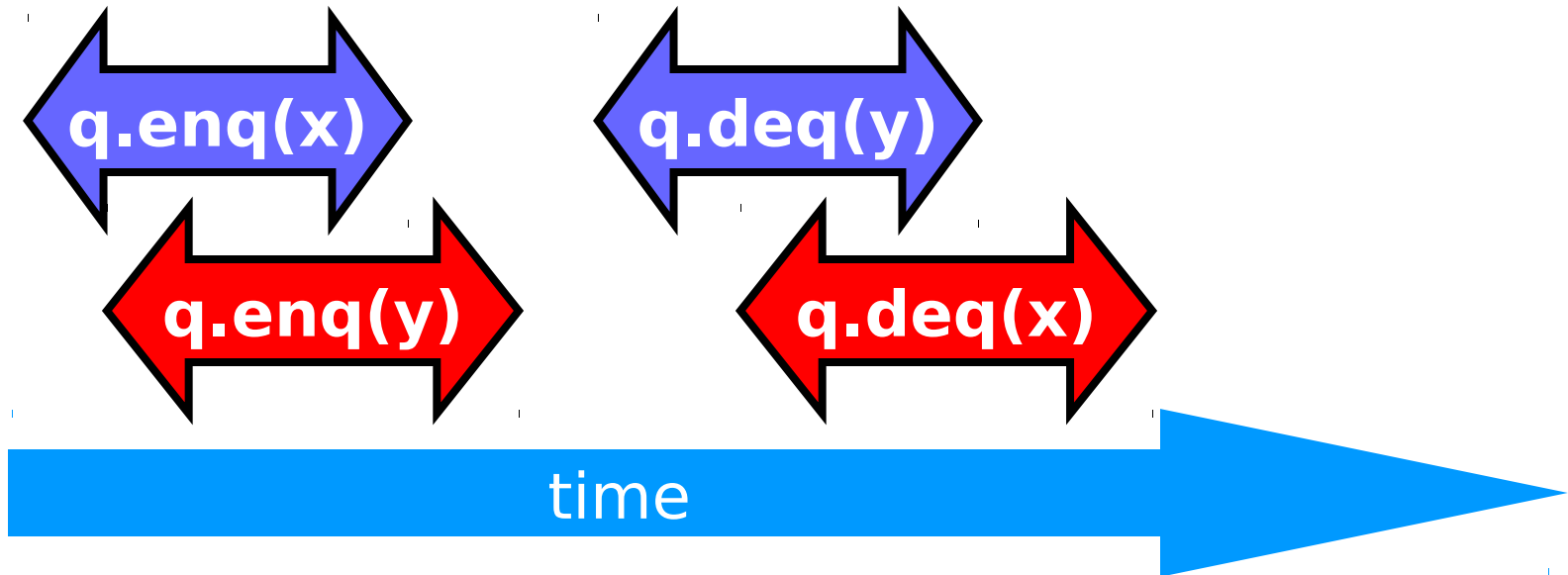
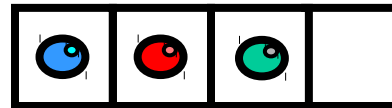


Example

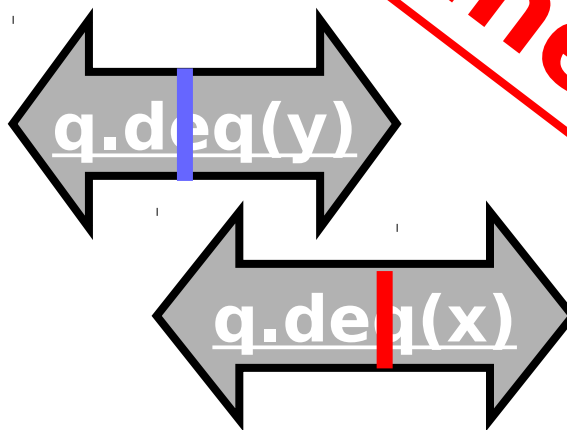
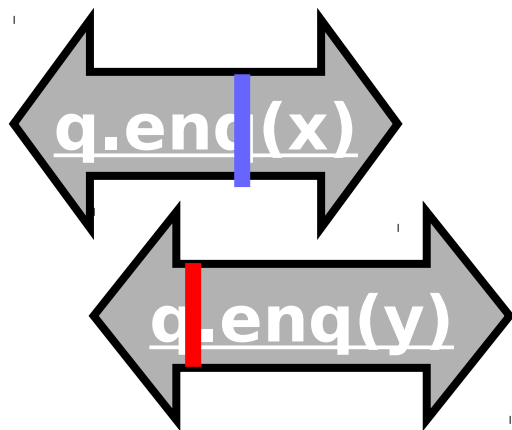
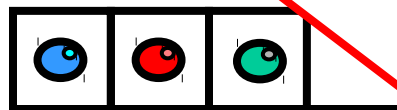




Example

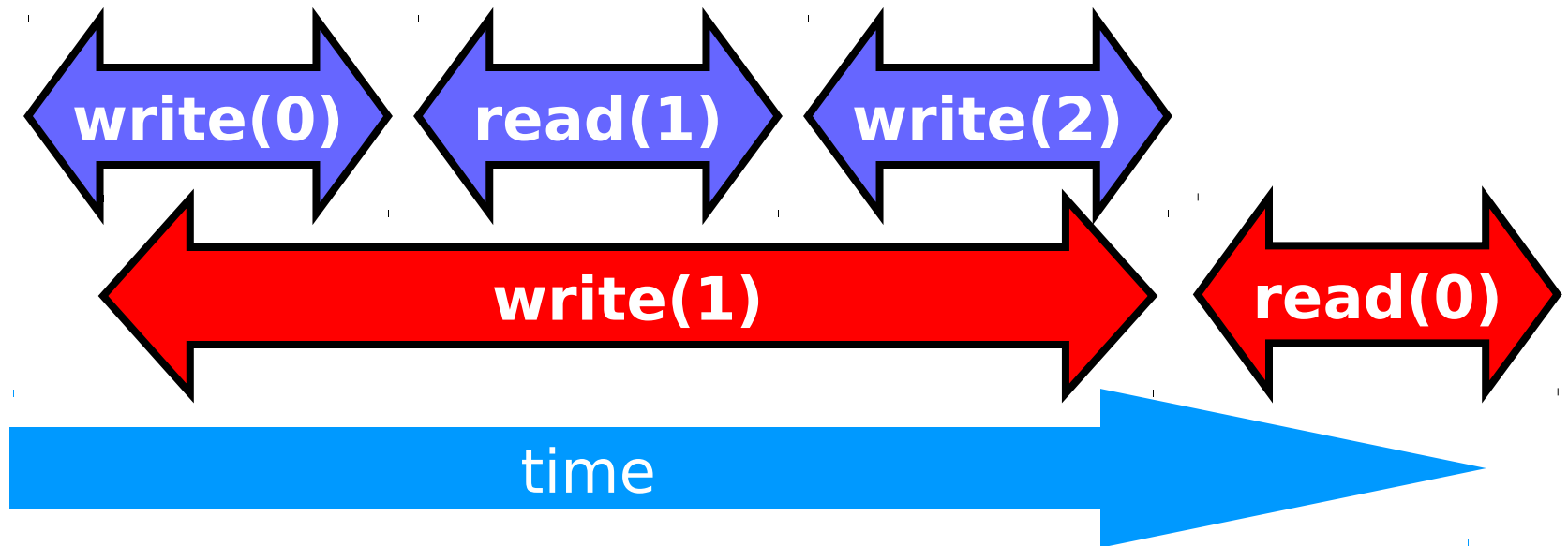


Comme ci Example
Comme ça

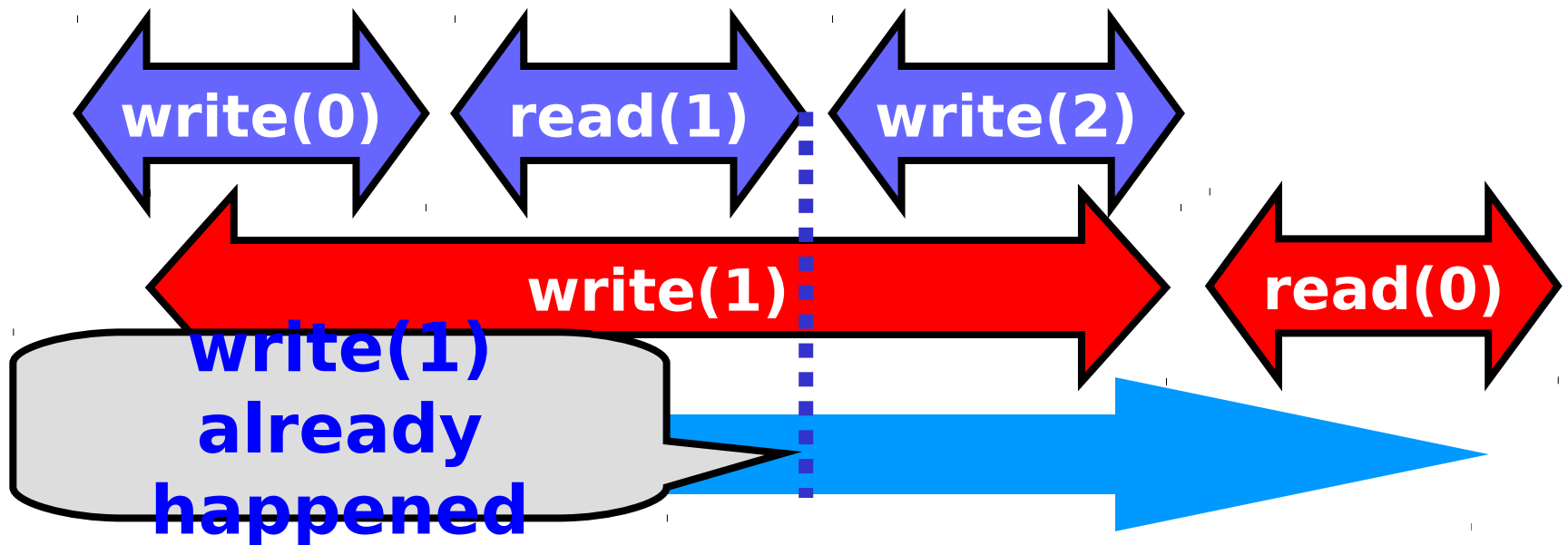


multiple orders
linearizable

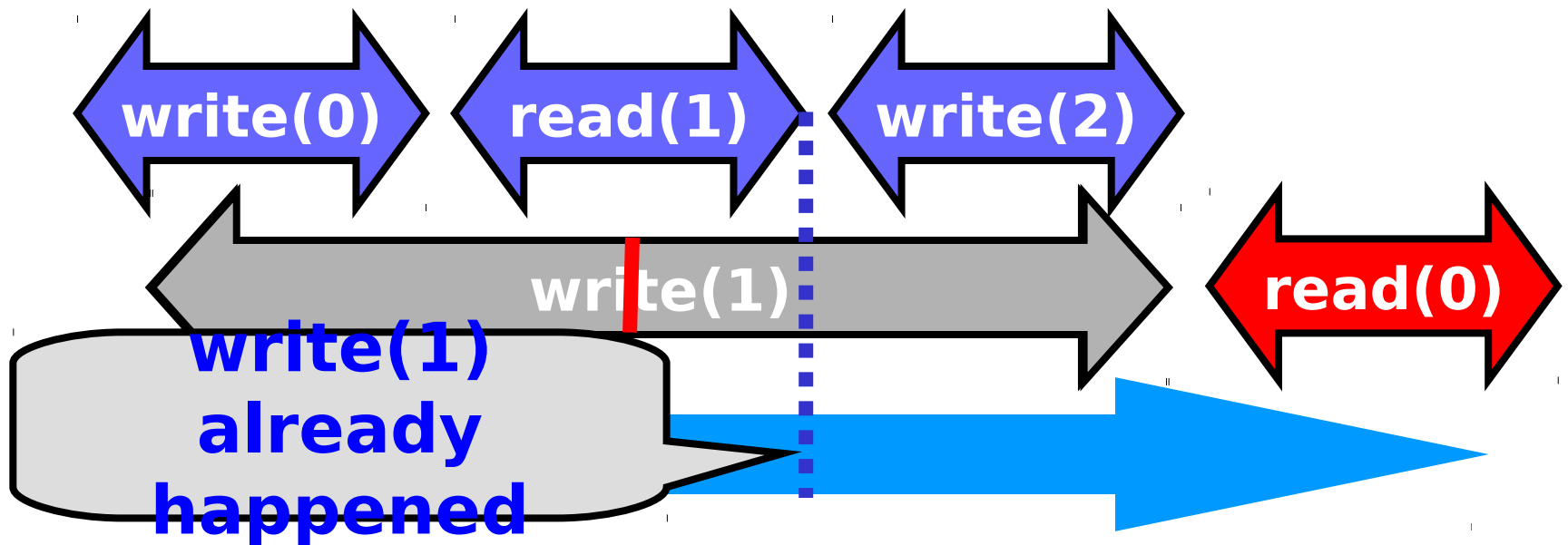
Read/Write Register Example



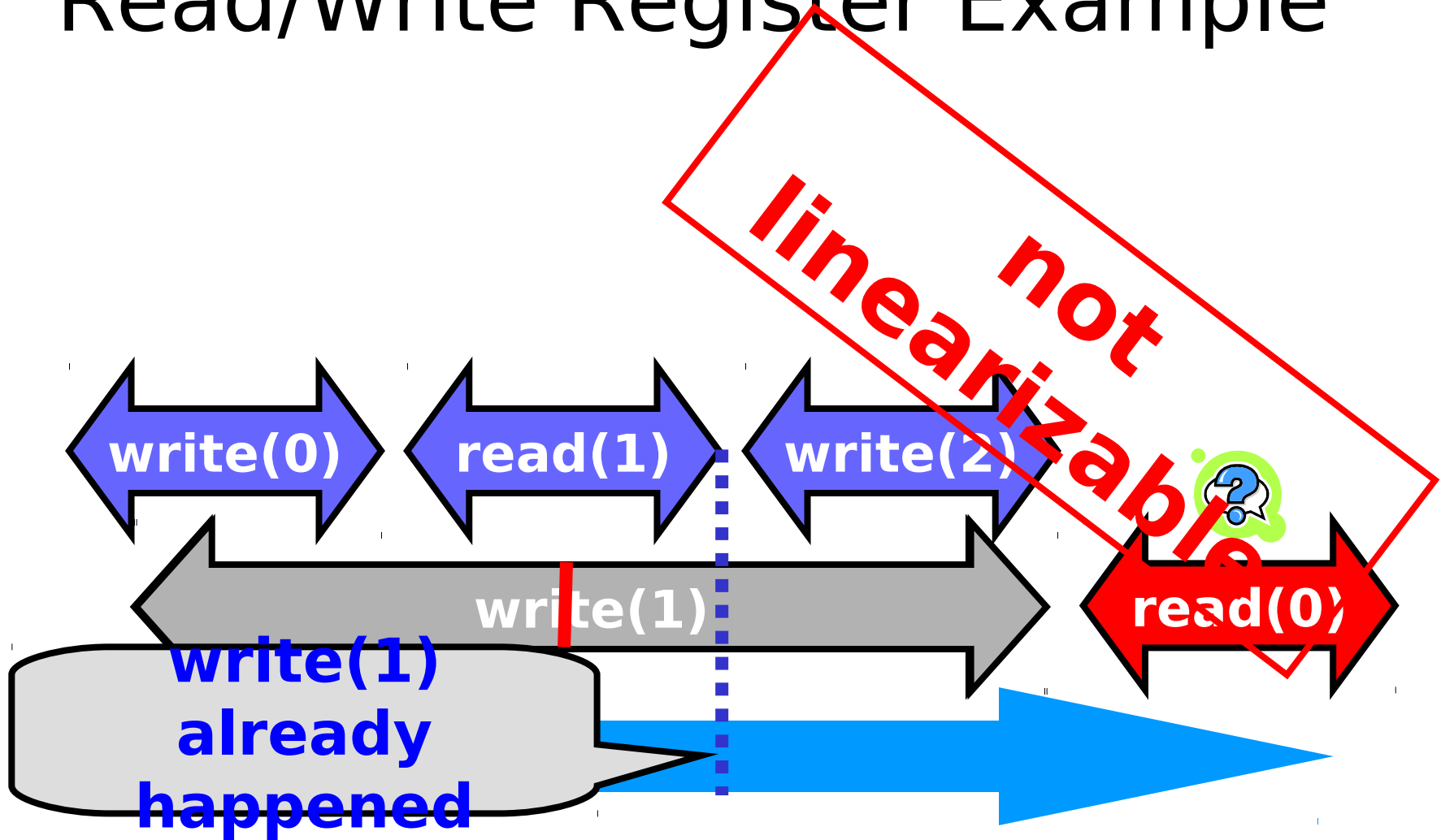
Read/Write Register Example



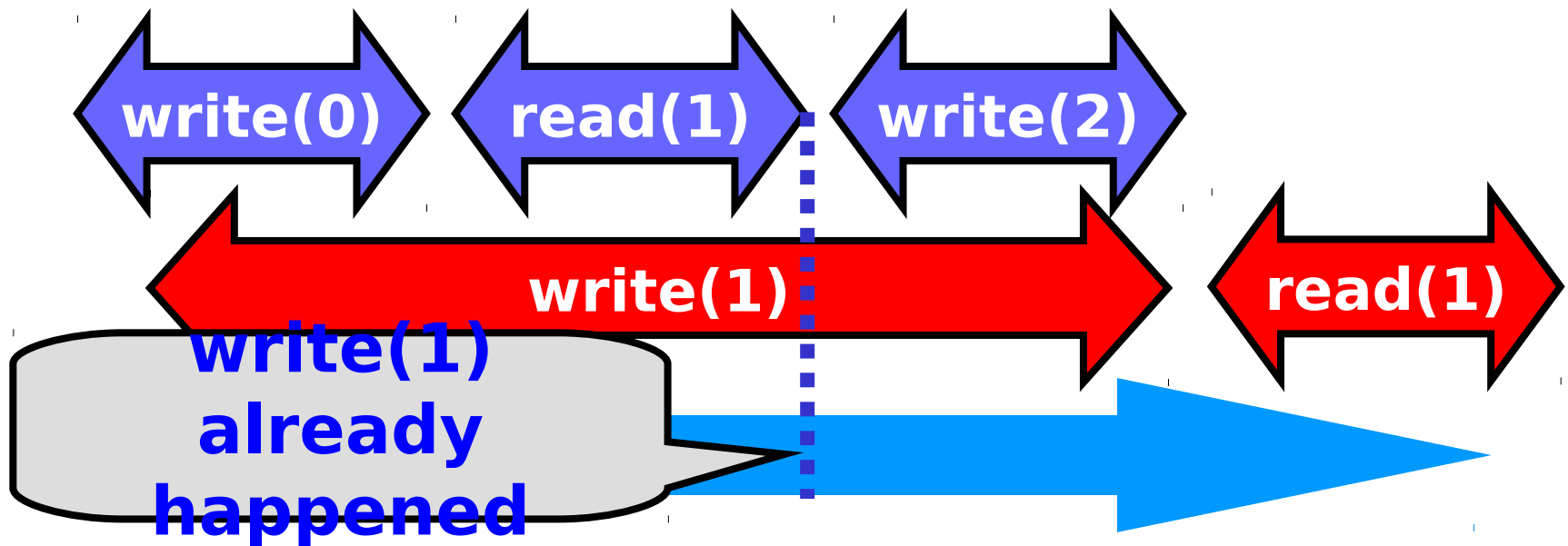
Read/Write Register Example



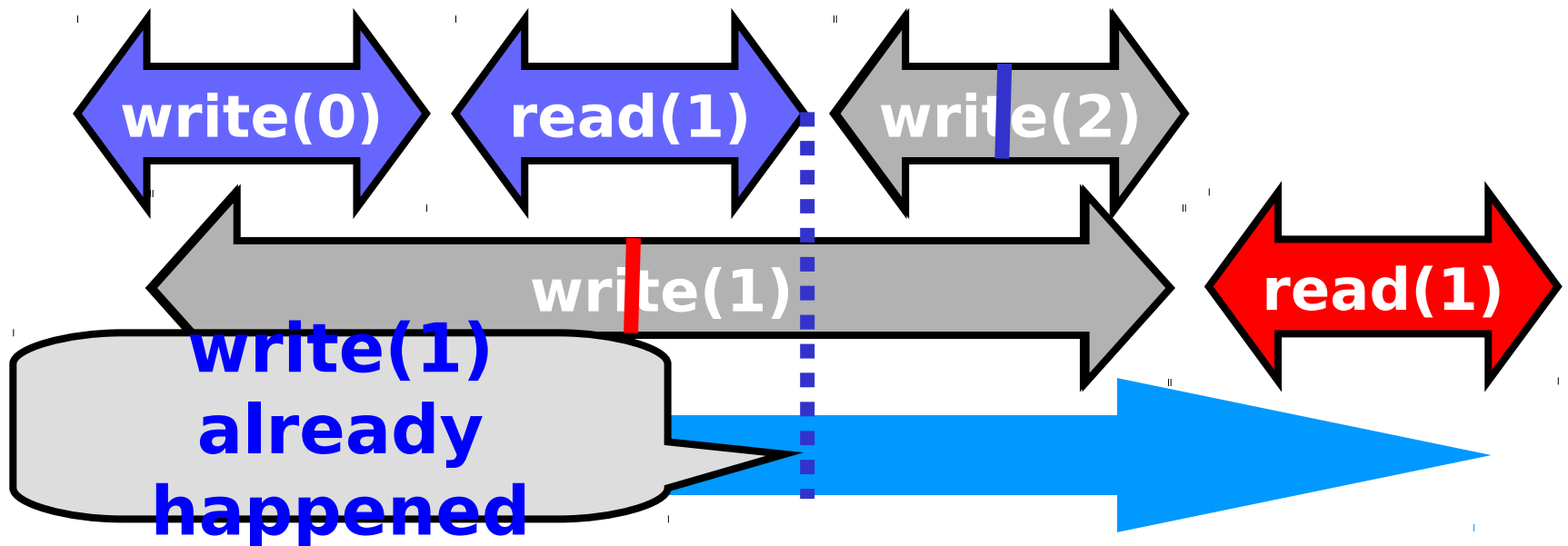
Read/Write Register Example



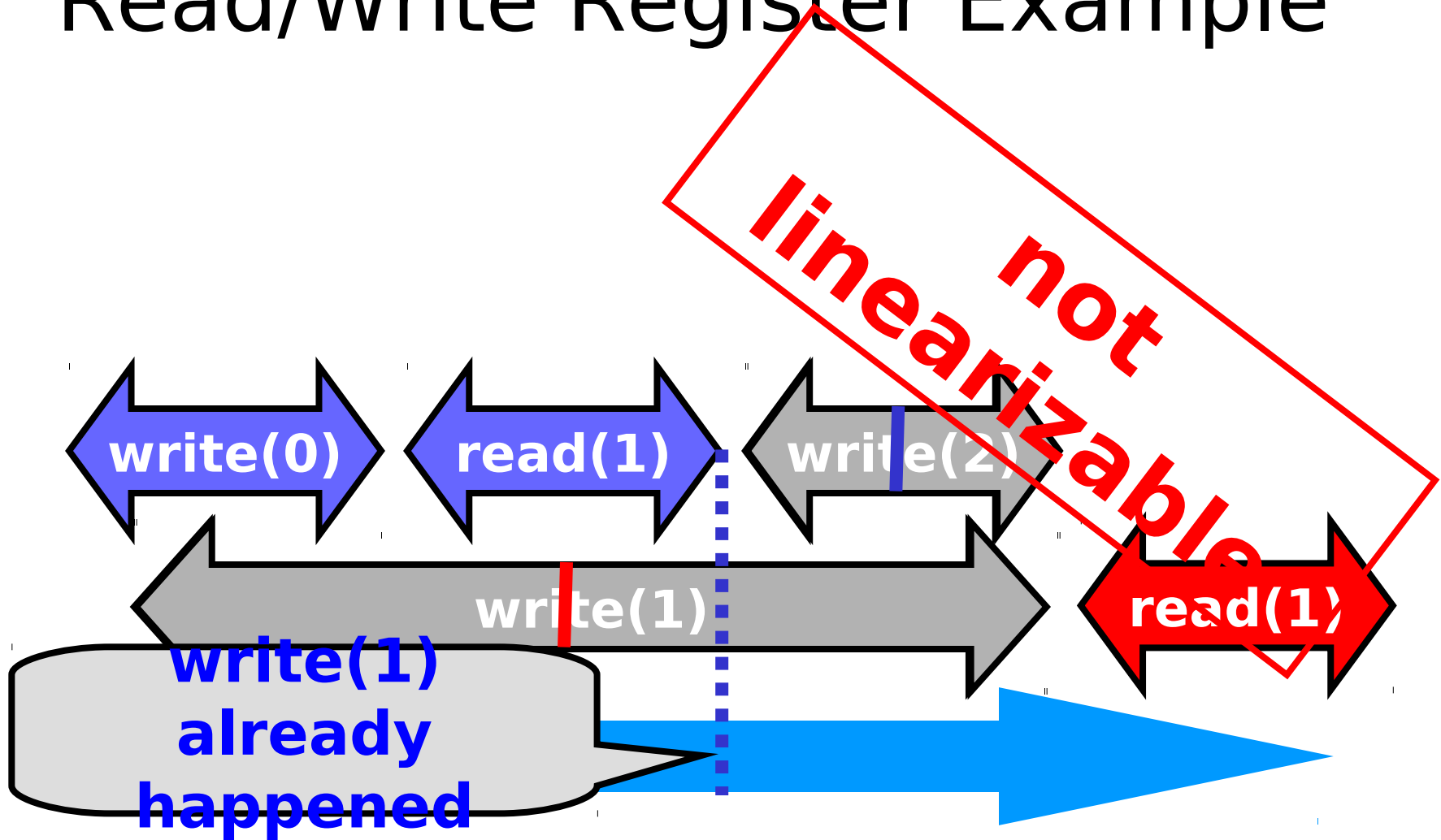
Read/Write Register Example



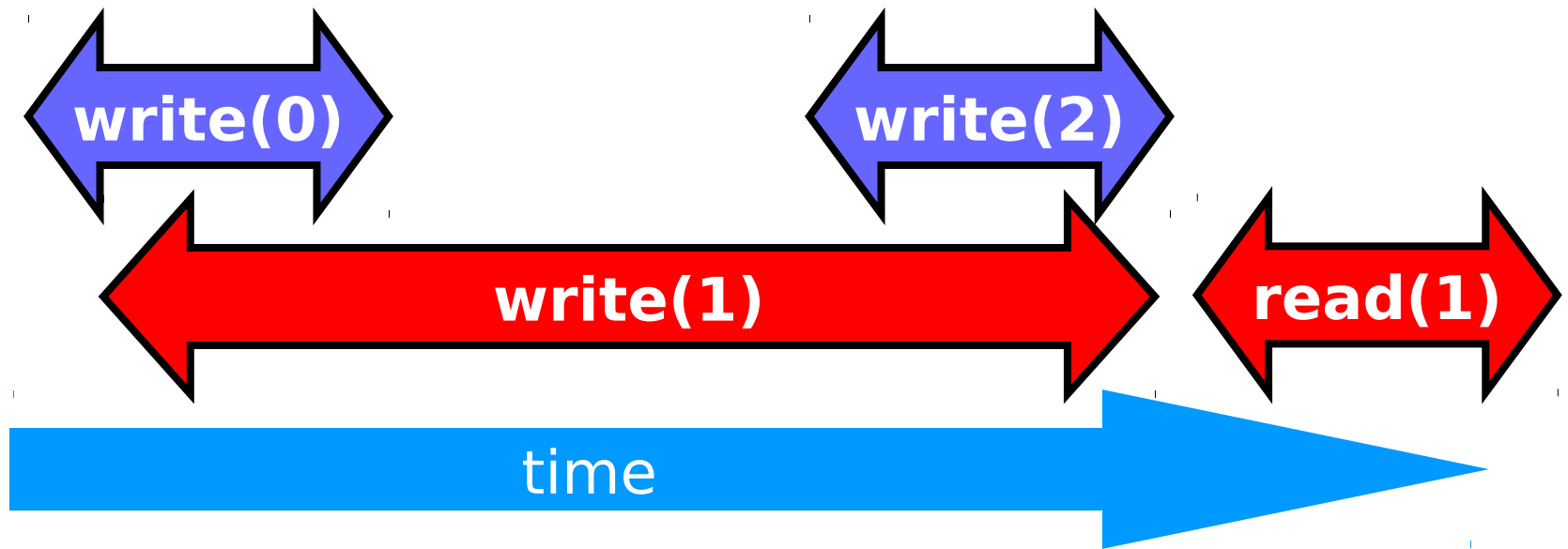
Read/Write Register Example



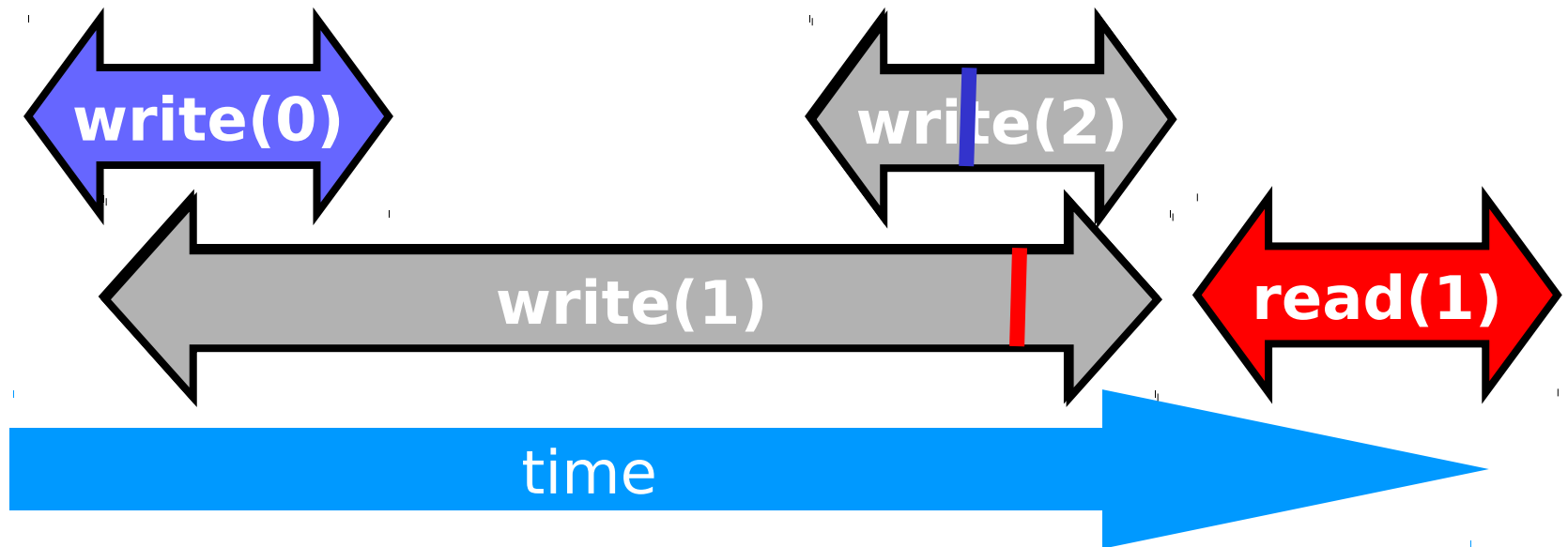
Read/Write Register Example



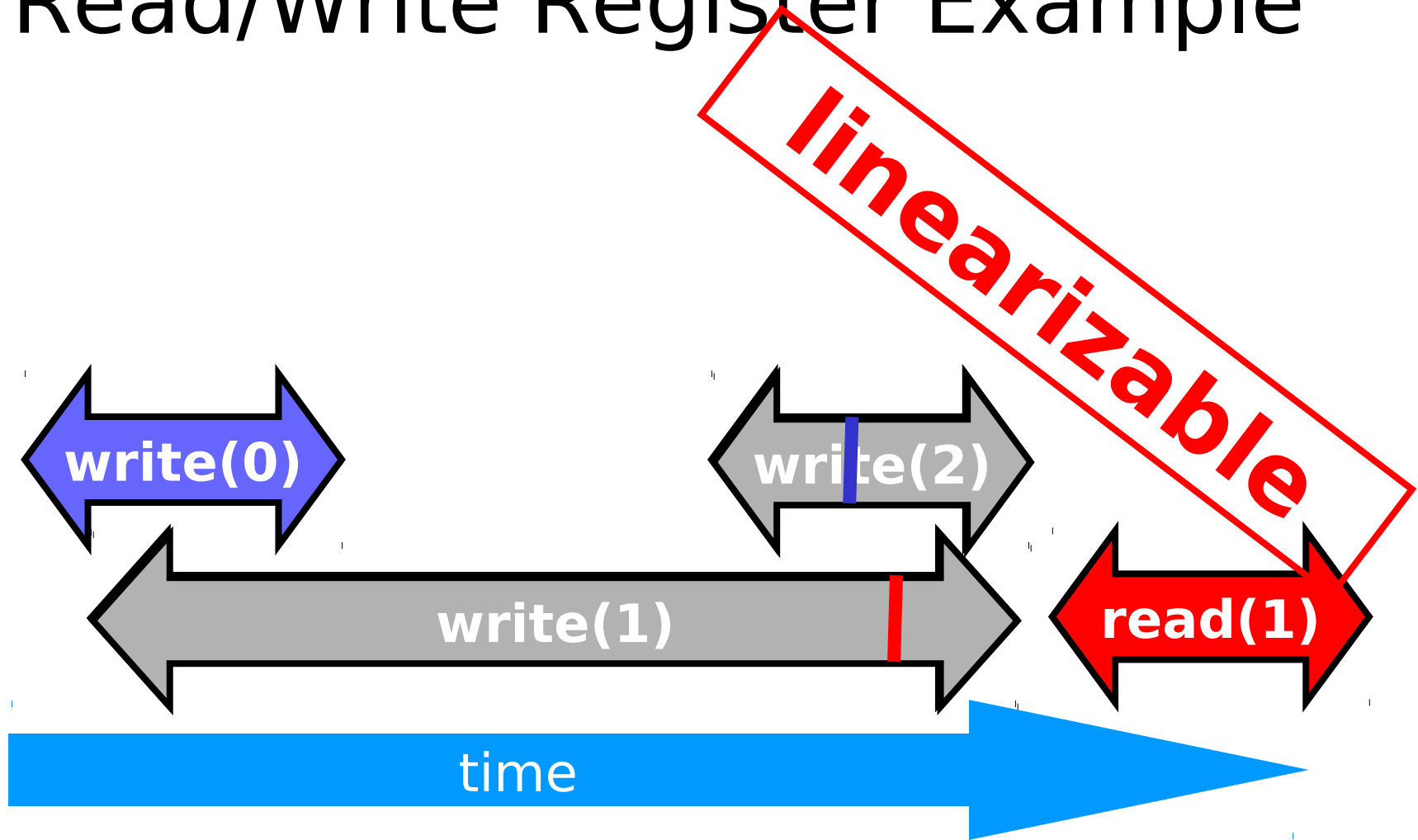
Read/Write Register Example



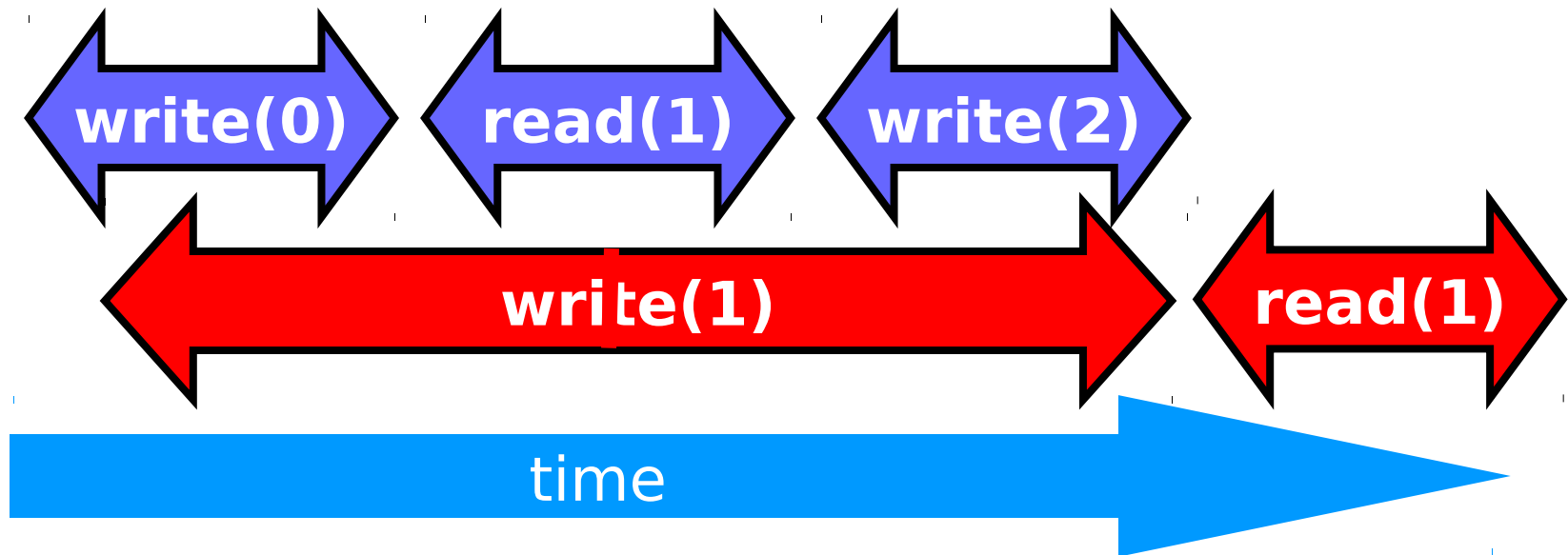
Read/Write Register Example



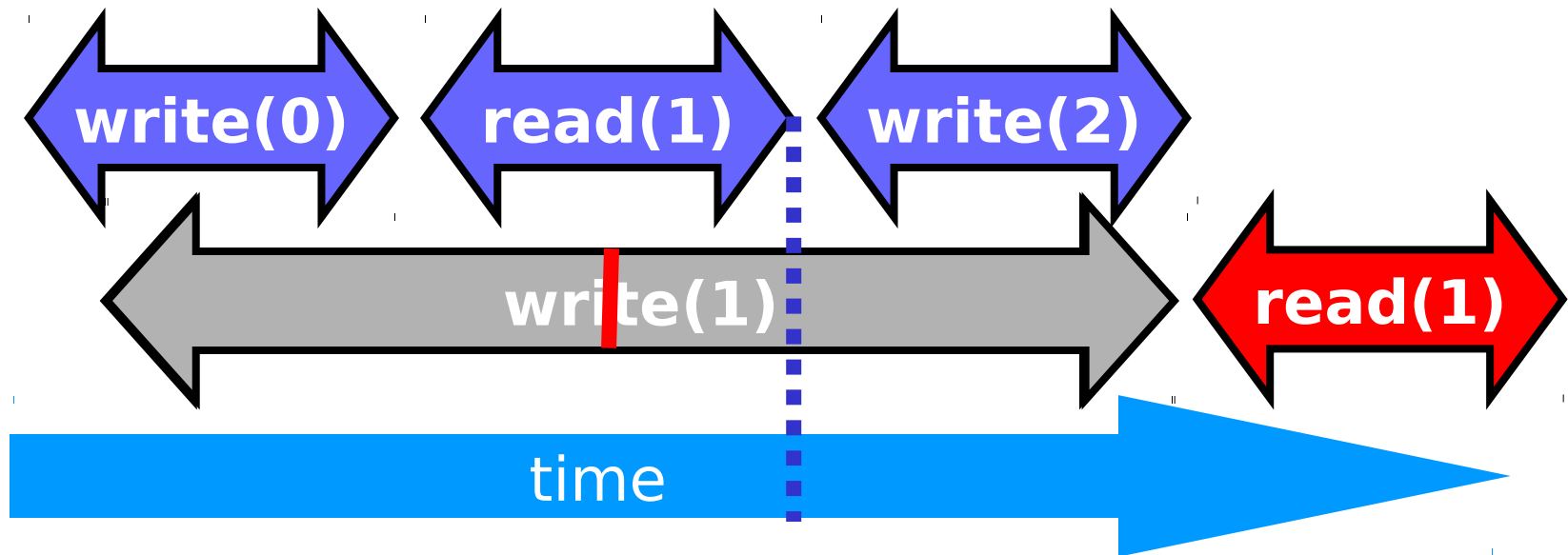
Read/Write Register Example



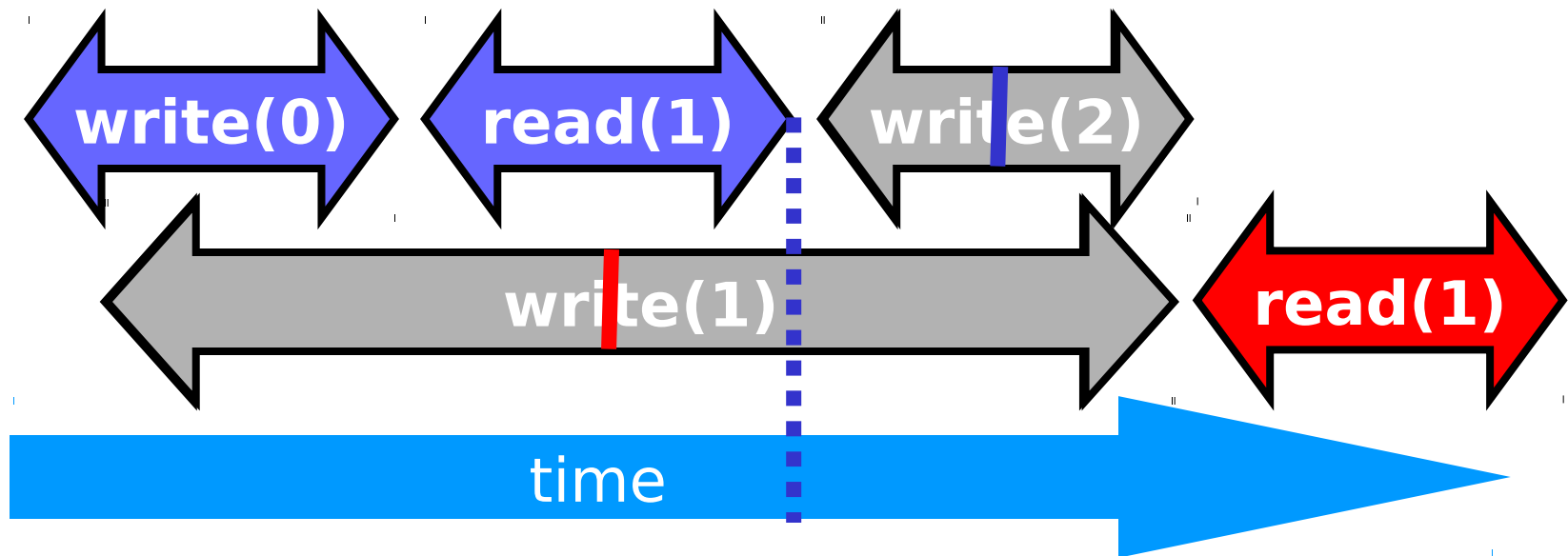
Read/Write Register Example



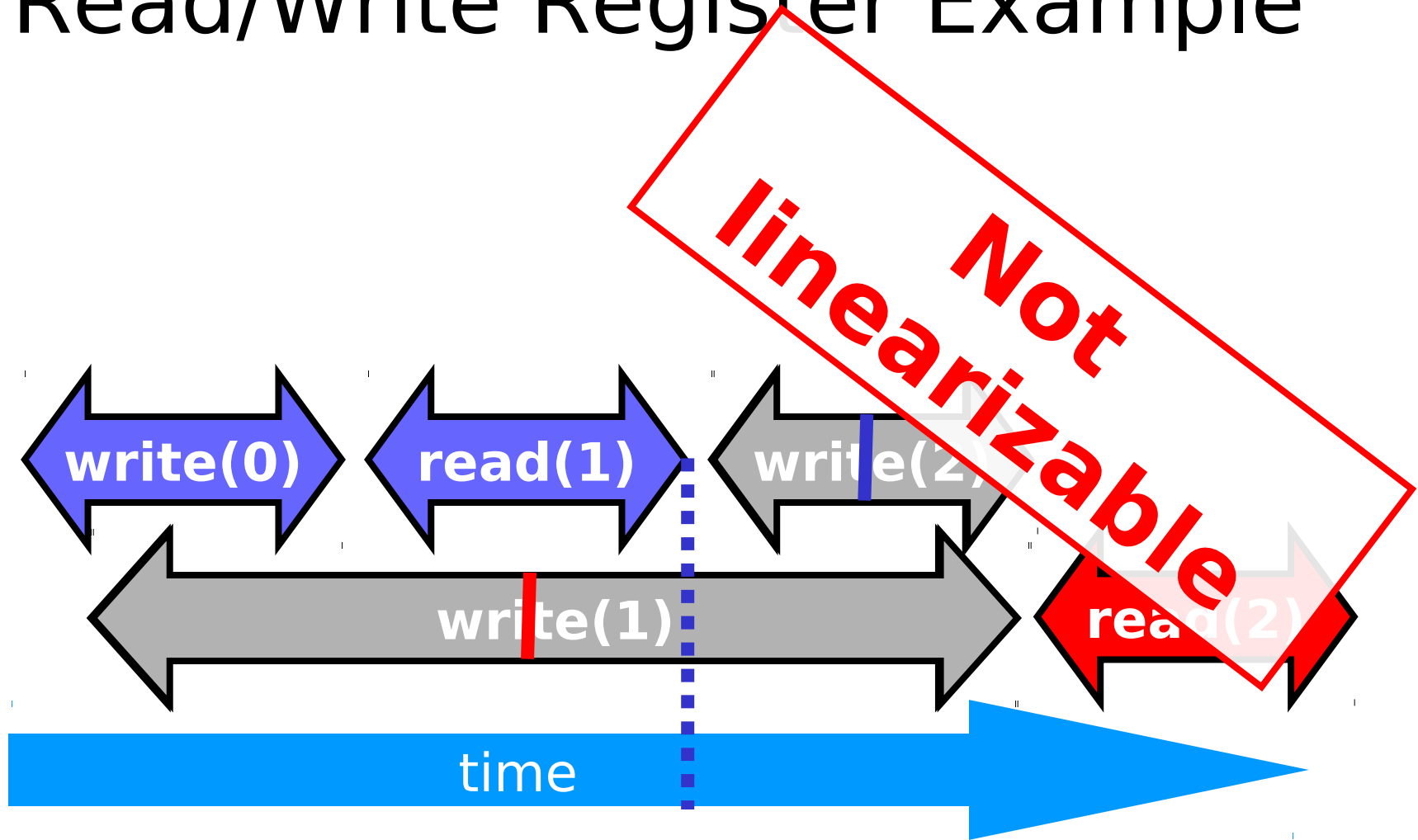
Read/Write Register Example



Read/Write Register Example



Read/Write Register Example



Talking About Executions

- Why?
 - Can't we specify the linearization point of each operation without describing an execution?
- Not Always
 - In some cases, linearization point depends on the execution

Formal Model of Executions

- Define precisely what we mean
 - Ambiguity is bad when intuition is weak
- Allow reasoning
 - Formal
 - But mostly informal
 - In the long run, actually more important
 - Ask me why!

Split Method Calls into Two Events

- Invocation
 - method name & args
 - **q.enq(x)**
- Response
 - result or exception
 - **q.enq(x)** returns **void**
 - **q.deq()** returns **x**
 - **q.deq()** throws **empty**

Invocation Notation

A q.enq(x)

Invocation Notation

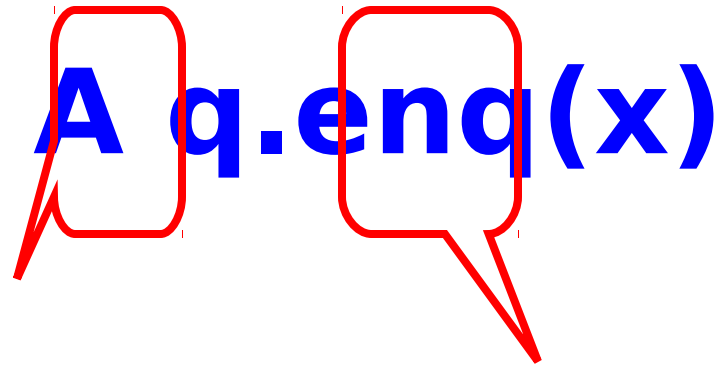
A q.enq(x)



thread

Invocation Notation

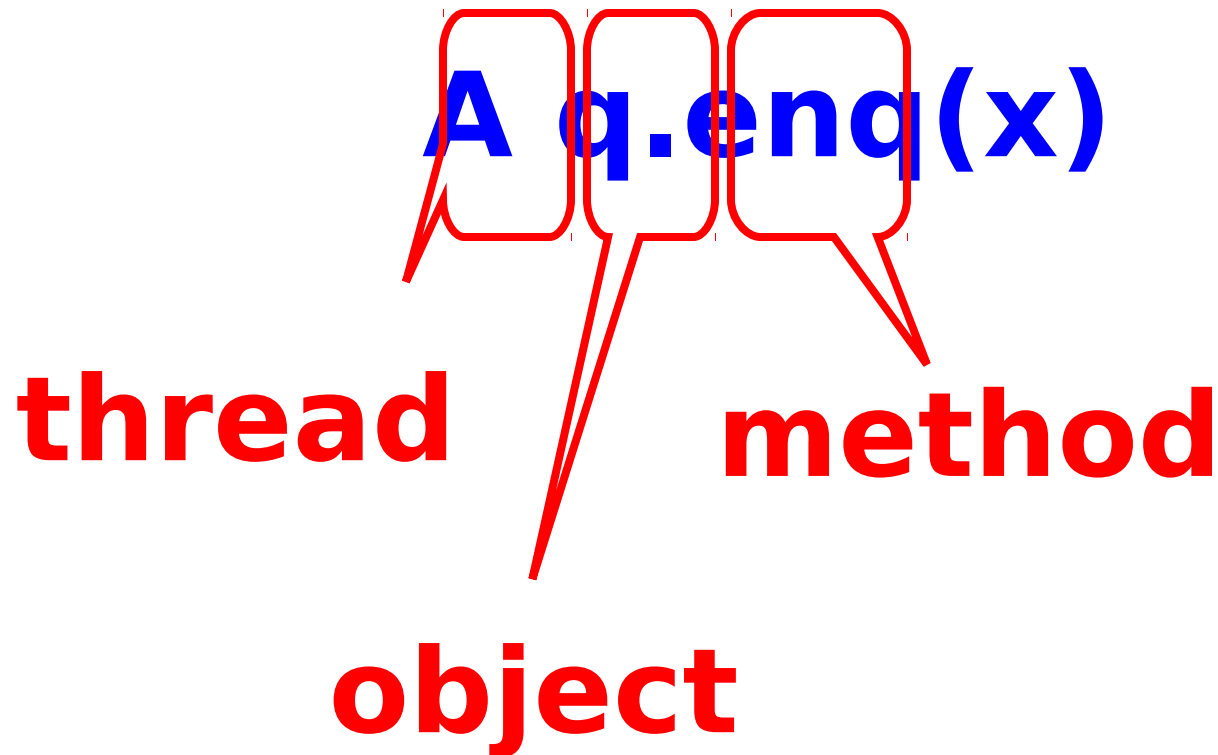
A q.end(x)

The diagram shows the text 'A q.end(x)' in blue. Two red callout boxes are positioned above the text. The first callout box is positioned over the letter 'A'. The second callout box is positioned over the text 'q.end(x)'. Both callout boxes have a tail pointing downwards towards the text they are highlighting.

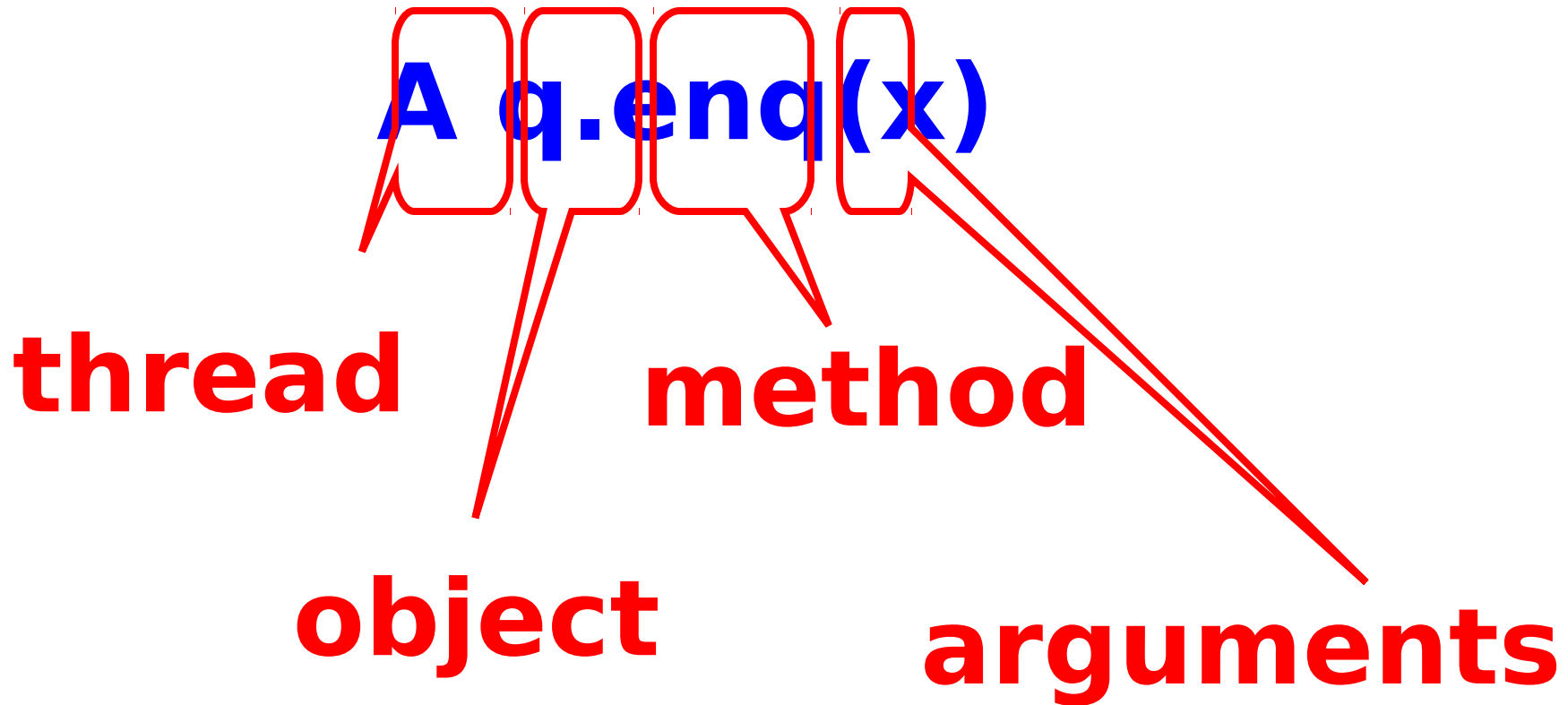
thread

method

Invocation Notation



Invocation Notation



Response Notation

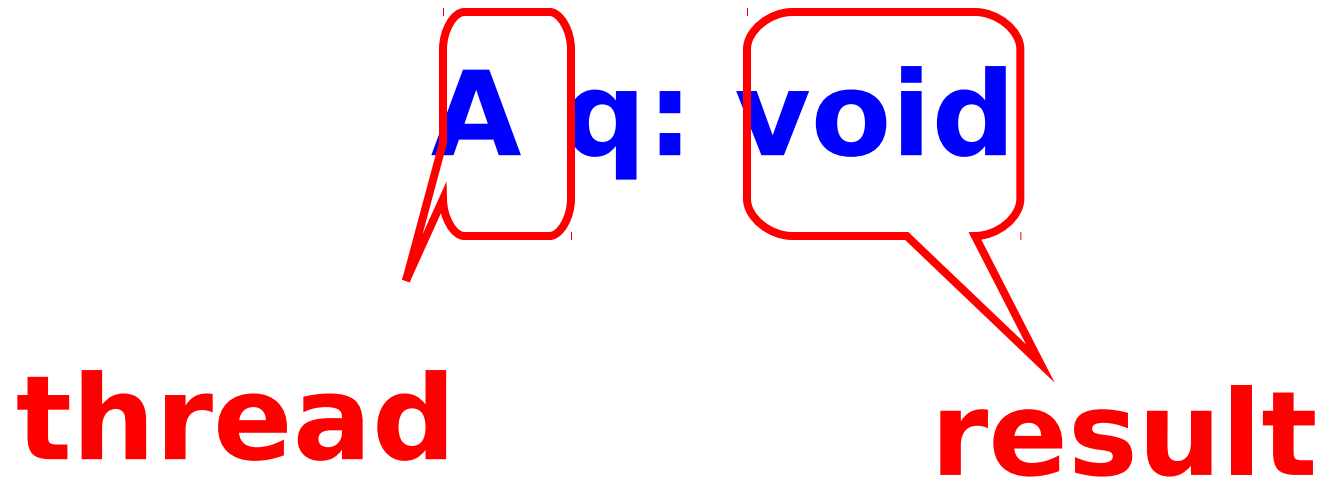
A q: void

Response Notation

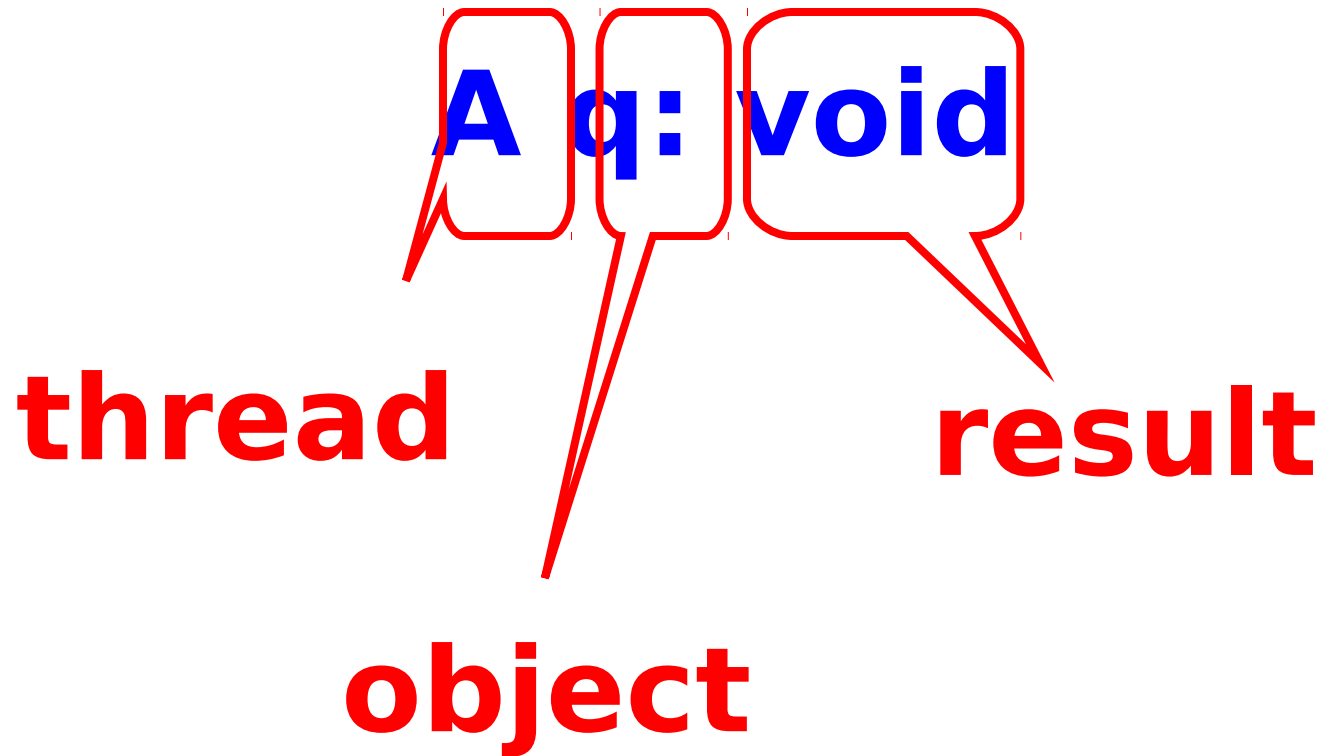
Aq: void

thread

Response Notation



Response Notation



Response Notation

Method is implicit

A q: void

thread

result

object

Response Notation

Method is implicit

A q: empty()

thread

exception

object

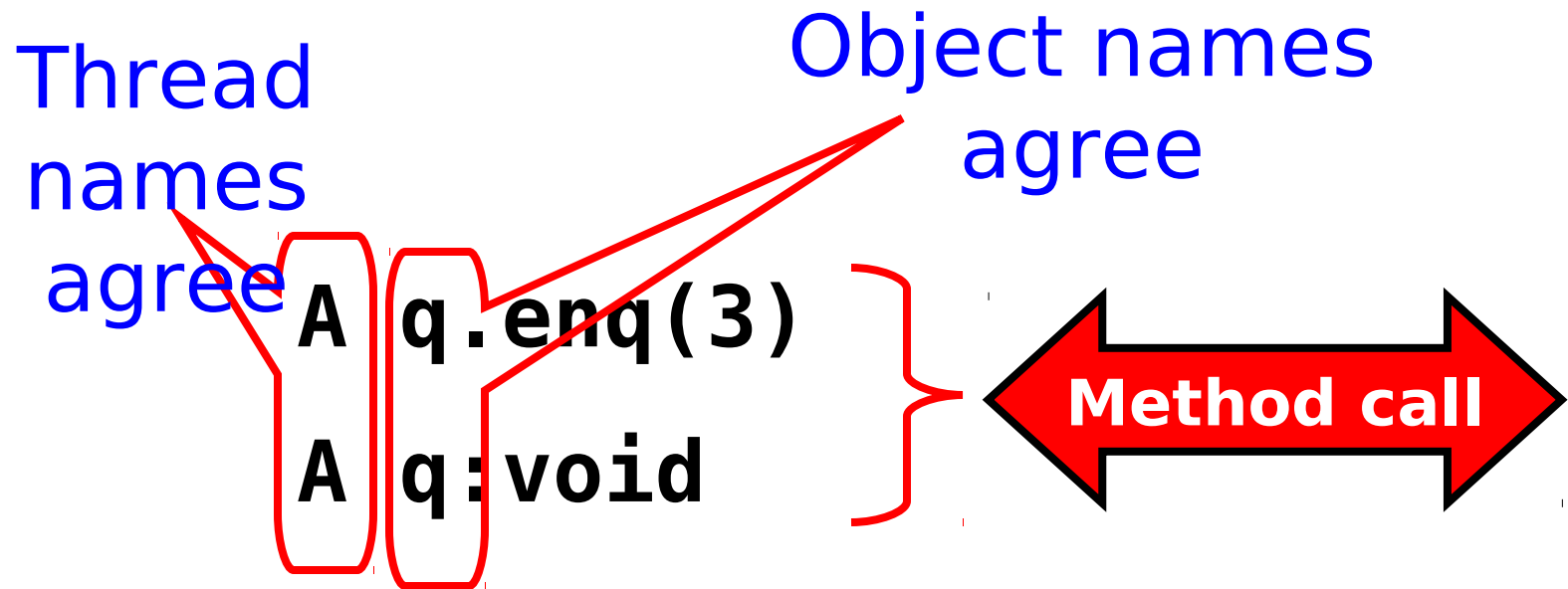
History - Describing an Execution

H = {
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3

**Sequence of
invocations and
responses**

Definition

- Invocation & response *match* if



Object Projections

H =

- A q.enq(3)**
- A q:void**
- B p.enq(4)**
- B p:void**
- B q.deq()**
- B q:3**

Object Projections

A q.enq(3)

A q:void

H|q =

B q.deq()

B q:3

Thread Projections

H =

- A q.enq(3)**
- A q:void**
- B p.enq(4)**
- B p:void**
- B q.deq()**
- B q:3**

Thread Projections

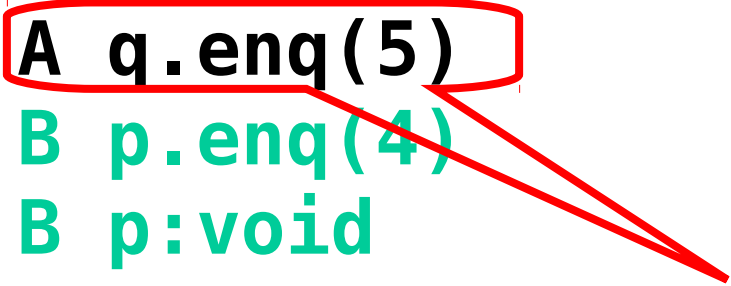
$H|B =$

- B** `p.enq(4)`
- B** `p:void`
- B** `q.deq()`
- B** `q:3`

Complete Subhistory

H =

```
A q.enqueue(3)
A q:void
A q.enqueue(5)
B p.enqueue(4)
B p:void
B q.dequeue()
B q:3
```

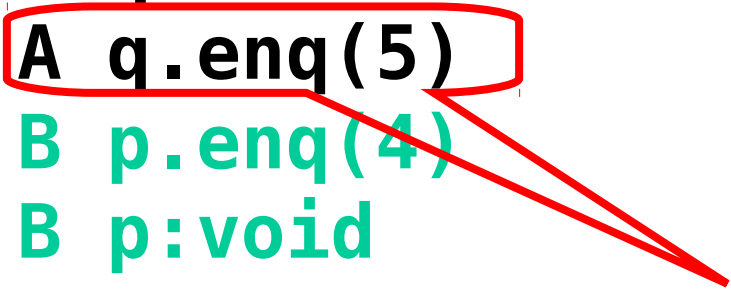


**An invocation is
pending if it has
no matching
response**

Complete Subhistory

H =

```
A q.enq(3)
A q:void
A q.enq(5)
B p.enq(4)
B p:void
B q.deq()
B q:3
```



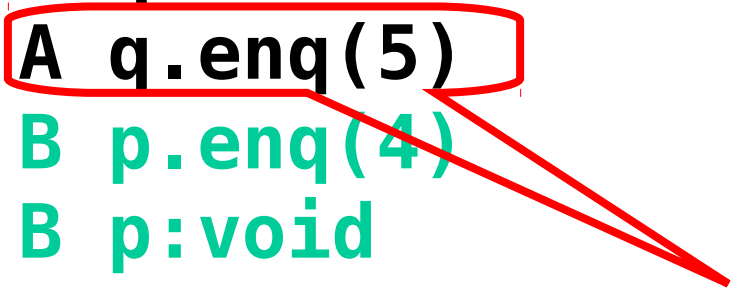
**May or may not
have taken
effect**

Complete Subhistory

H =

- A q.enq(3)
- A q:void
- A q.enq(5)**
- B p.enq(4)
- B p:void
- B q.deq()
- B q:3

discard pending invocations



Complete Subhistory

A q.enq(3)
A q:void

Complete(H) = B p.enq(4)
B p:void
B q.deq()
B q:3

Sequential Histories

A q.enq(3)

A q:void

B p.enq(4)

B p:void

B q.deq()

B q:3

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

B p.enq(4)

B p:void

B q.deq()

B q:3

A q:enq(5)

match

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

B q:3

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

B q:3

match

A q:enq(5)

Sequential Histories

A q.enq(3)

A q:void

match

B p.enq(4)

B p:void

match

B q.deq()

B q:3

match

A q:enq(5)

**Final pending
invocation OK**

Sequential Histories

A q.enq(3)
A q:void

B p.enq(4)
B p:void

B q.deq()
B q:3

A q:enq(5)

Method calls of
do different threads of
do not interleave

match
match
match
Final pending
invocation OK

Well-Formed Histories

H=

- A q.enq(3)**
- B p.enq(4)**
- B p:void**
- B q.deq()**
- A q:void**
- B q:3**

Well-Formed Histories

**Per-thread
projections
sequential**

H=
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3

H | B=
B p.enq(4)
B p:void
B q.deq()
B q:3

Well-Formed Histories

**Per-thread
projections
sequential**

H=
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3

H | B=
B p.enq(4)
B p:void
B q.deq()
B q:3

H | A=
A q.enq(3)
A q:void

Equivalent Histories

Threads see the same thing in both

$$\left\{ \begin{array}{l} H|A = G|A \\ H|B = G|B \end{array} \right.$$

H=

```
A q.enq(3)
B p.enq(4)
B p:void
B q.deq()
A q:void
B q:3
```

G=

```
A q.enq(3)
A q:void
B p.enq(4)
B p:void
B q.deq()
B q:3
```

Sequential Specifications

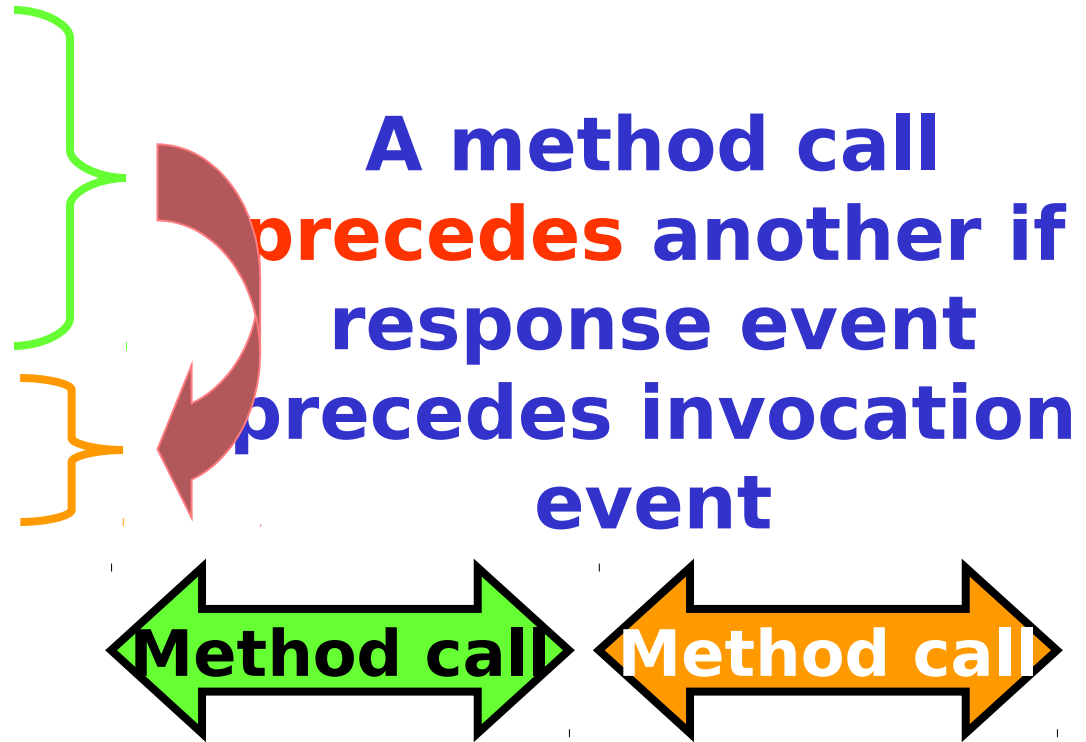
- A sequential specification is some way of telling whether a
 - Single-thread, single-object history
 - Is legal
- For example:
 - Pre and post-conditions
 - But plenty of other techniques exist ...

Legal Histories

- A sequential (multi-object) history H is legal if
 - For every object x
 - $H|x$ is in the sequential spec for x

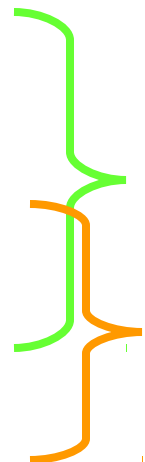
Precedence

A q.enq(3)
B p.enq(4)
B p.void
A q:void
B q.deq()
B q:3



Non-Precedence

A q.enq(3)
B p.enq(4)
B p.void
B q.deq()
A q:void
B q:3

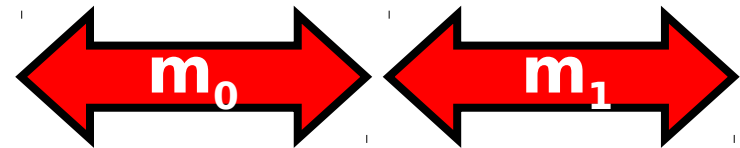


Some method calls **overlap** one another



Notation

- Given
 - _ History **H**
 - _ method executions **m₀** and **m₁** in **H**
- We say $m_0 \rightarrow_H m_1$, if
 - _ **m₀** precedes **m₁**
- Relation $m_0 \rightarrow_H m_1$ is a
 - _ Partial order
 - _ Total order if **H** is sequential



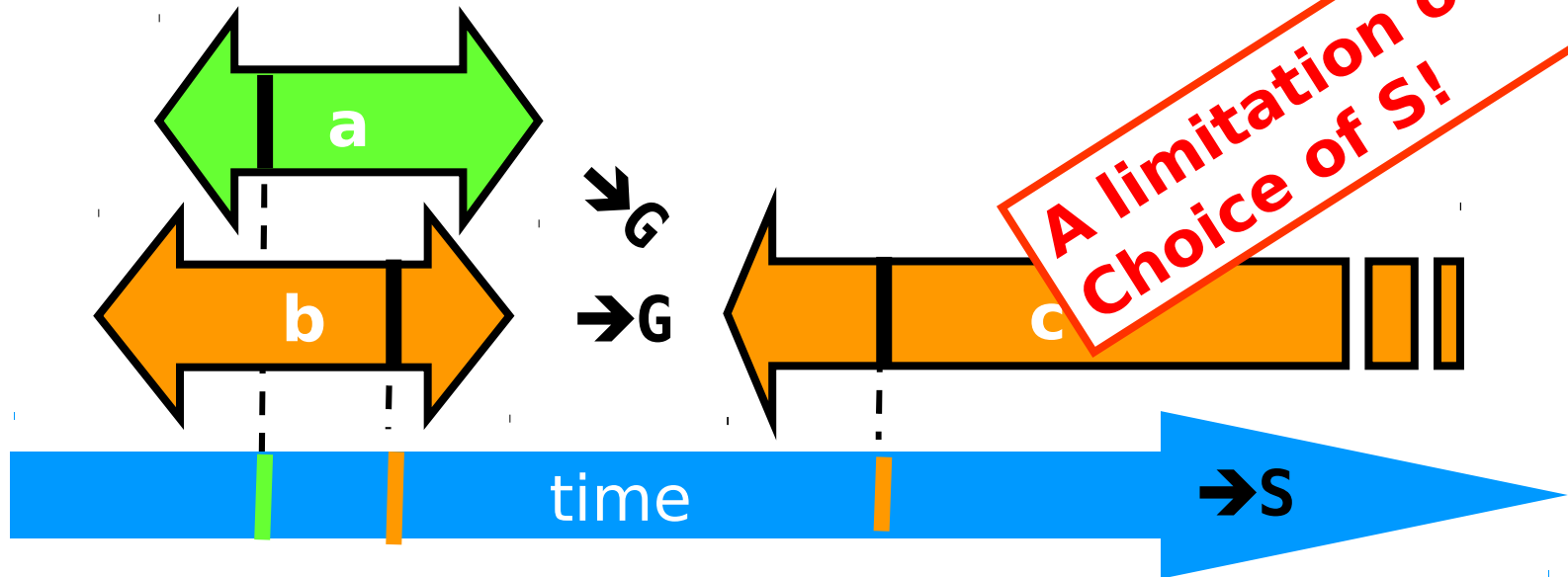
Linearizability

- History H is ***linearizable*** if it can be extended to \mathbf{G} by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that \mathbf{G} is equivalent to
 - Legal sequential history \mathbf{S}
 - where $\rightarrow_{\mathbf{G}} \subset \rightarrow_{\mathbf{S}}$

What is $\rightarrow_G \subset \rightarrow_S$

$$\rightarrow_G = \{a \rightarrow c, b \rightarrow c\}$$

$$\rightarrow_S = \{a \rightarrow b, a \rightarrow c, b \rightarrow c\}$$



Remarks

- Some pending invocations
 - Took effect, so keep them
 - Discard the rest
- Condition $\rightarrow_G \subset \rightarrow_S$
 - Means that **S** respects “real-time order” of **G**

Example

A q.enq(3)

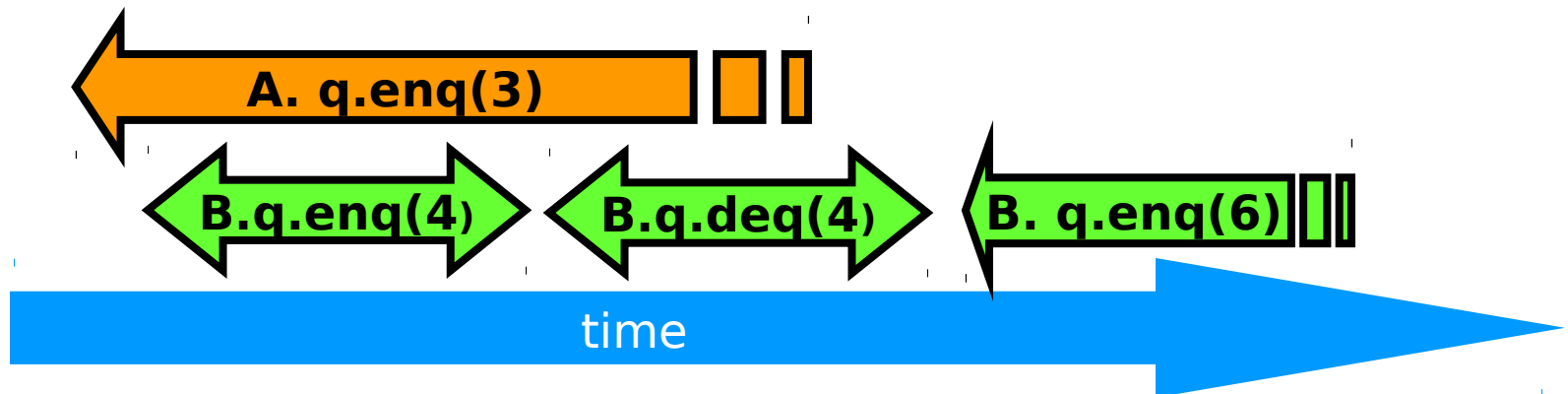
B q.enq(4)

B q:void

B q.deq()

B q:4

B q:enq(6)



Example

A q.enq(3)

B q.enq(4)

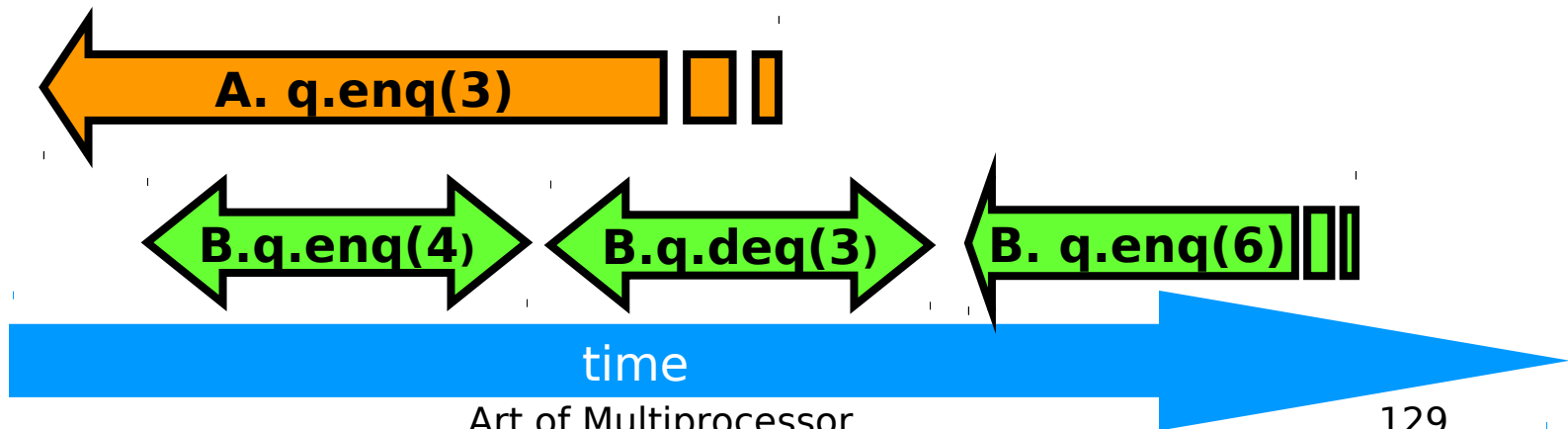
B q:void

B q.deq()

B q:4

B q:enq(6)

**Complete this
pending
invocation**



Example

A q.enq(3)

B q.enq(4)

B q:void

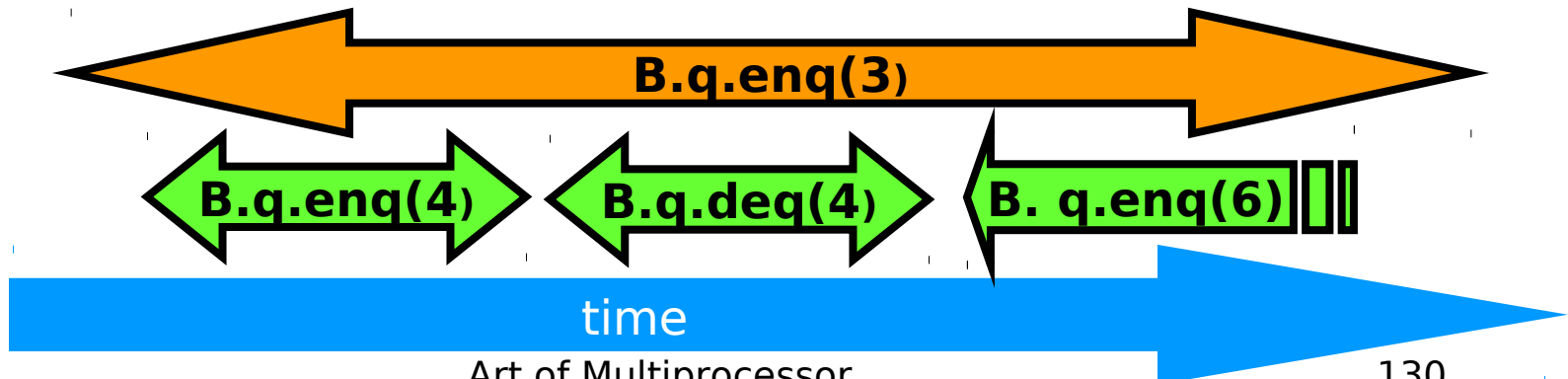
B q.deq()

B q:4

B q:enq(6)

A q:void

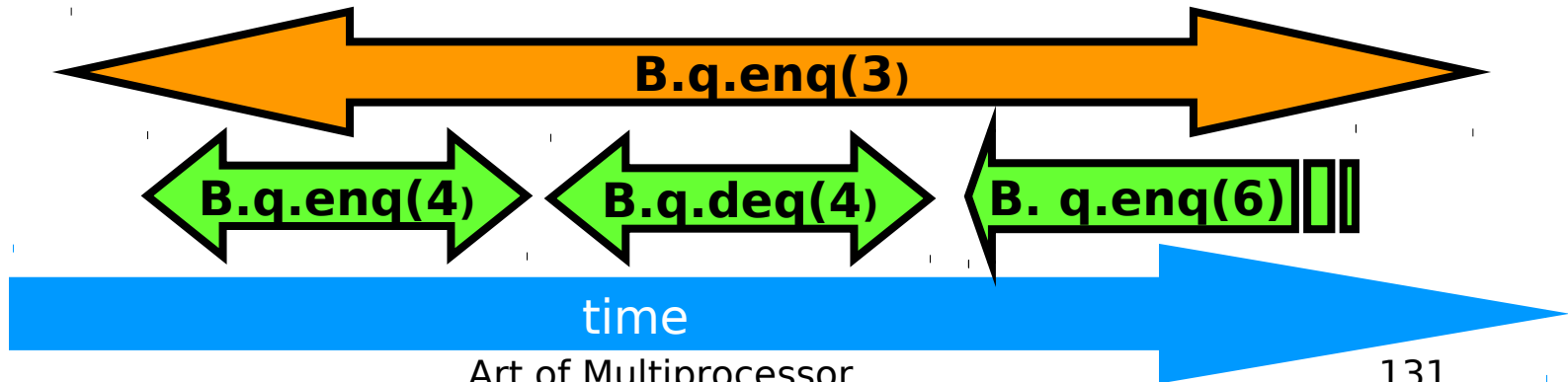
Complete this pending invocation



Example

```
A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
B q:enq(6)
A q:void
```

discard this
one



Example

A q.enq(3)

B q.enq(4)

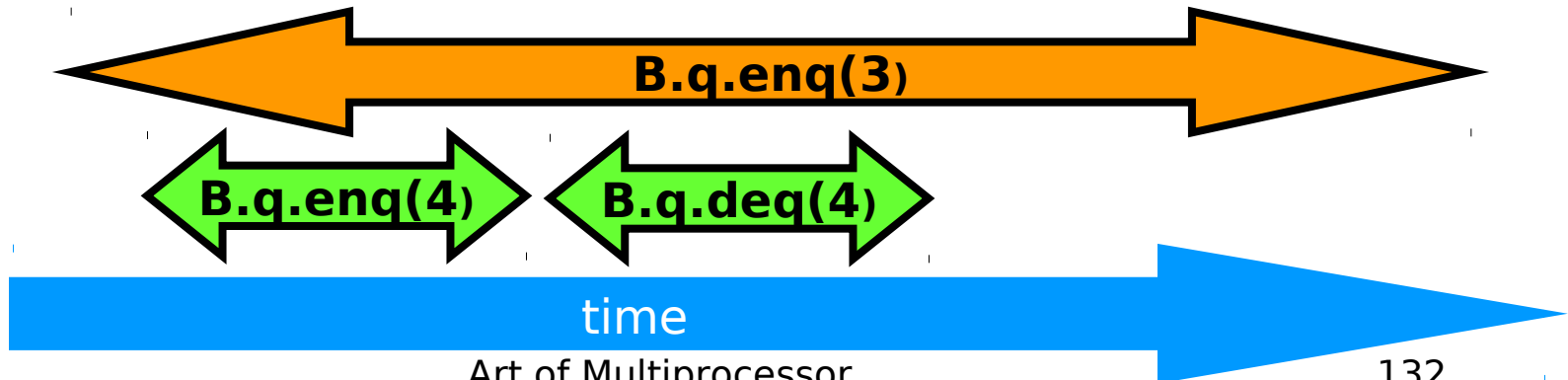
B q:void

B q.deq()

B q:4

A q:void

discard this
one



Example

A q.enq(3)

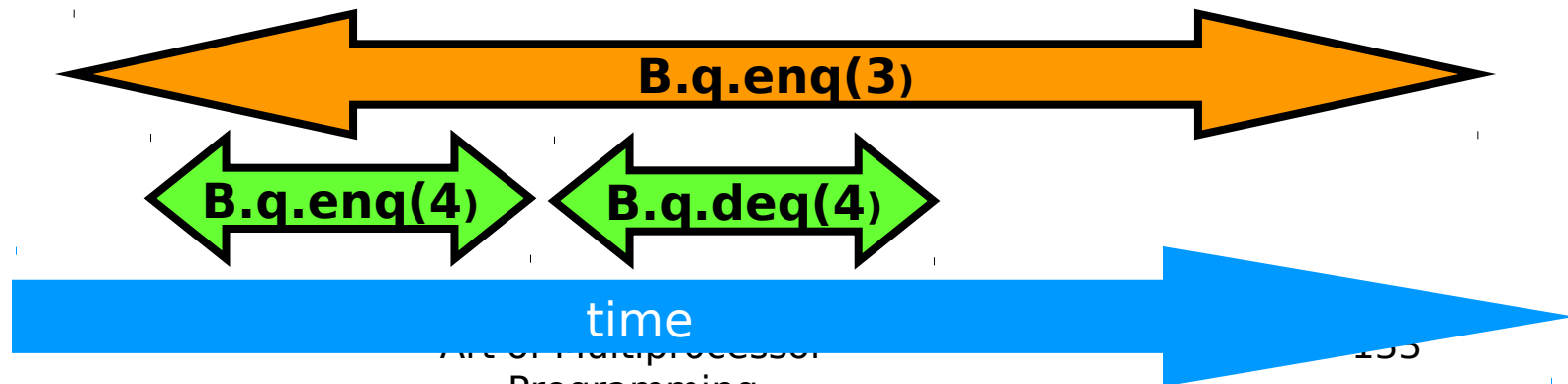
B q.enq(4)

B q:void

B q.deq()

B q:4

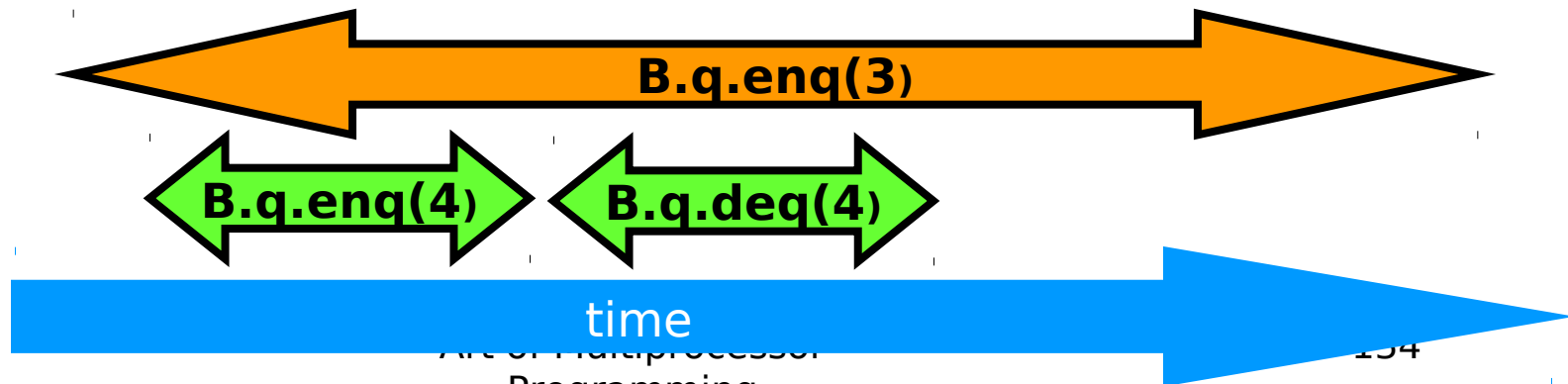
A q:void



Example

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4

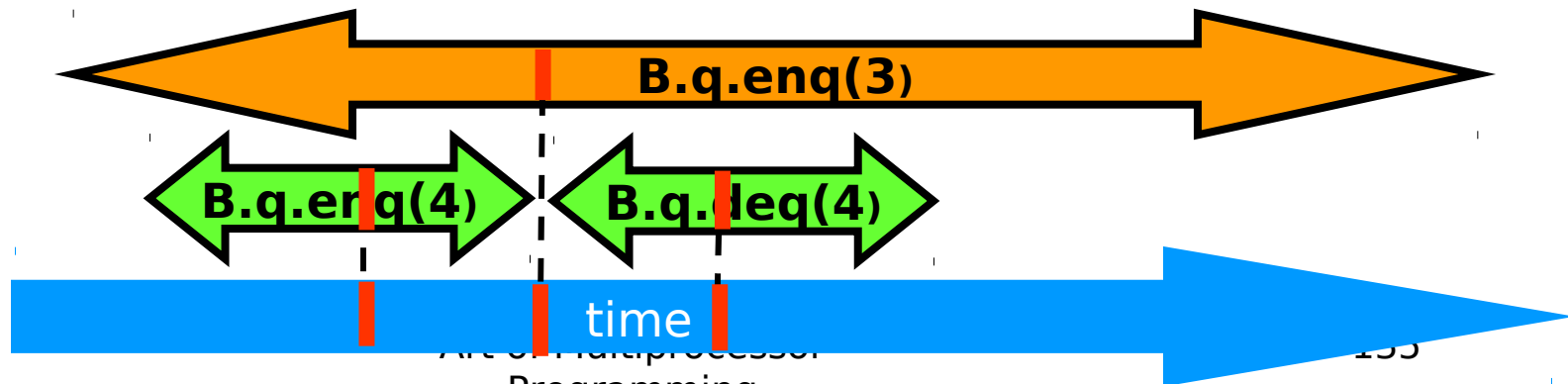


Example

Equivalent sequential history

A q.enq(3)
B q.enq(4)
B q:void
B q.deq()
B q:4
A q:void

B q.enq(4)
B q:void
A q.enq(3)
A q:void
B q.deq()
B q:4



Concurrency

- How much concurrency does linearizability allow?
- When must a method invocation block?

Concurrency

- Focus on ***total*** methods
 - Defined in every state
- Example:
 - `deq()` that throws `Empty` exception
 - Versus `deq()` that waits ...
- Why?
 - Otherwise, blocking unrelated to synchronization

Concurrency

- **Question:** When does linearizability require a method invocation to block?
- **Answer:** never.
- Linearizability is *non-blocking*

Non-Blocking Theorem

If method invocation

A q.inv(...)

is pending in history H, then there exists a response

A q:res(...)

such that

H + A q:res(...)

is linearizable

Proof

- Pick linearization S of H
- If S already contains
 - Invocation $A \ q.\mathit{inv}(\dots)$ and response,
 - Then we are done.
- Otherwise, pick a response such that
 - $S + A \ q.\mathit{inv}(\dots) + A \ q:\mathit{res}(\dots)$
 - Possible because object is *total*.

Composability Theorem

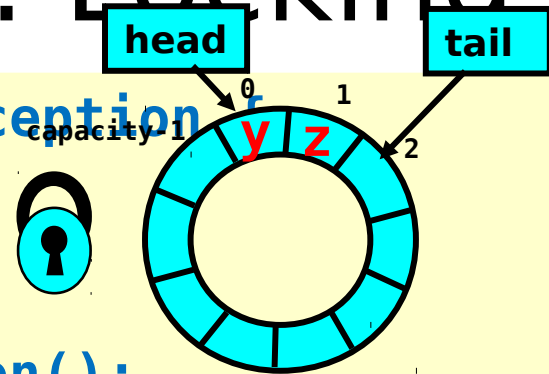
- History H is linearizable if and only if
 - For every object x
 - $H|x$ is linearizable
- We care about objects only!
 - (Materialism?)

Why Does Composability ?Matter

- Modularity
- Can prove linearizability of objects in isolation
- Can compose independently-implemented objects

Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {
    lock.lock();
    try {
        if (tail == head)
            throw new EmptyException();
        T x = items[head % items.length];
        head++;
        return x;
    } finally {
        lock.unlock();
    }
}
```



Reasoning About Linearizability: Locking

```
public T deq() throws EmptyException {  
    lock.lock();  
    try {  
        if (tail == head)  
            throw new EmptyException();  
        T x = items[head % items.length];  
        head++;  
        return x;  
    } finally {  
        lock.unlock();  
    }  
}
```

Linearization points
are when locks are
released

More Reasoning: Lock-free

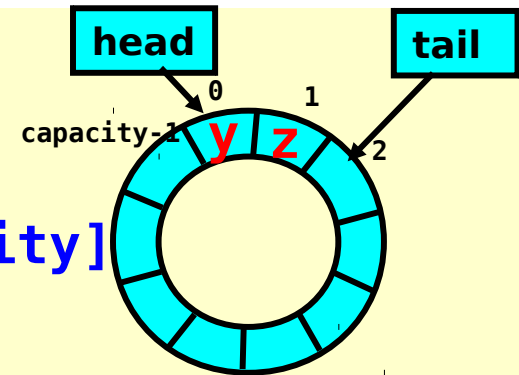
```
public class LockFreeQueue {
```

```
    int head = 0, tail = 0;  
    items = (T[]) new Object[capacity]
```

```
    public void enq(Item x) {  
        while (tail-head == capacity); // busy-wait  
        items[tail % capacity] = x; tail++;  
    }
```

```
    public Item deq() {  
        while (tail == head); // busy-wait  
        Item item = items[head % capacity]; head++;  
        return item;  
    }
```

```
}}
```



More Reasoning

Remember that there is only one enqueuer and only one dequeuer

```
public class Deque {
    int head, tail = 0;
    Item[] items = new Item[capacity];

    void enq(Item x) {
        while (tail - head == capacity); // busy-wait
        items[tail % capacity] = x;
        tail++;
    }

    public Item deq() {
        while (tail == head); // busy-wait
        Item item = items[head % capacity];
        return item;
        head++;
    }
}
```

Linearization order is order head and tail fields modified

tail++;

head++;

Strategy

- Identify one atomic step where method “happens”
 - Critical section
 - Machine instruction
- Doesn't always work
 - Might need to define several different steps for a given method

Linearizability: Summary

- Powerful specification tool for shared objects
- Allows us to capture the notion of objects being “atomic”
- Don't leave home without it

Alternative: Sequential Consistency

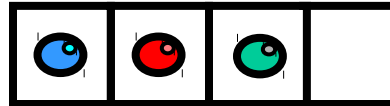
- History H is **Sequentially Consistent** if it can be extended to G by
 - Appending zero or more responses to pending invocations
 - Discarding other pending invocations
- So that G is equivalent to a
 - Legal sequential history S **Differs from linearizability**

~~- Where $\rightarrow G \subset \rightarrow S$~~

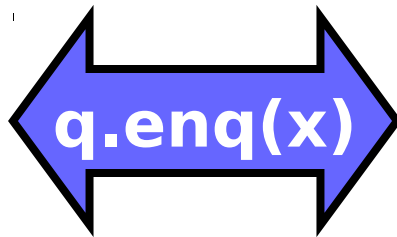
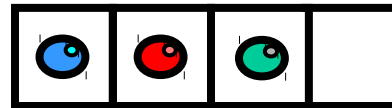
Alternative: Sequential Consistency

- No **need to preserve** real-time order
 - Cannot **re-order operations done by the same thread**
 - Can **re-order non-overlapping operations done by different threads**
- Often used to describe multiprocessor memory architectures

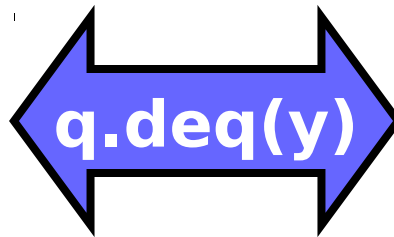
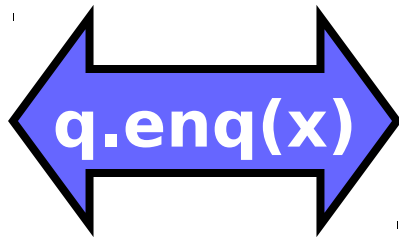
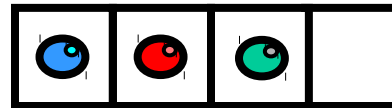
Example



Example

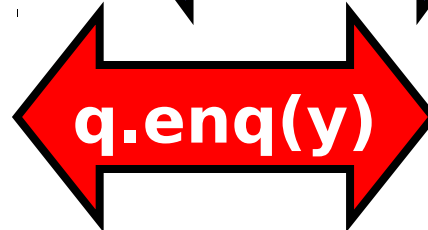
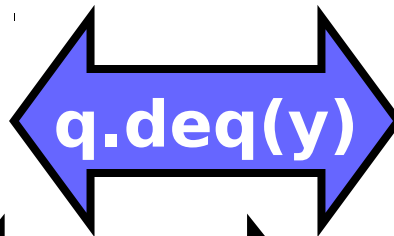
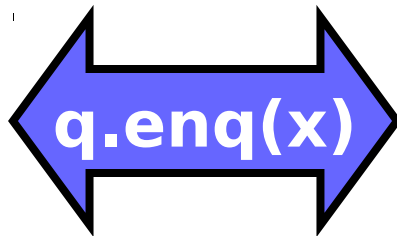
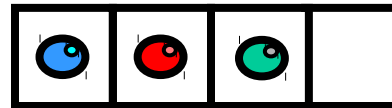


Example



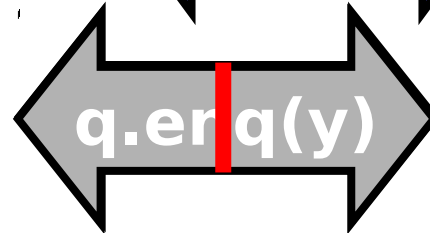
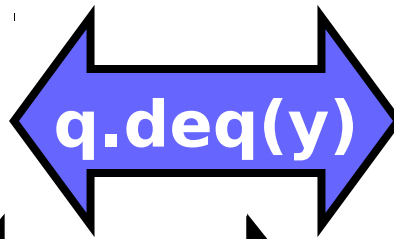
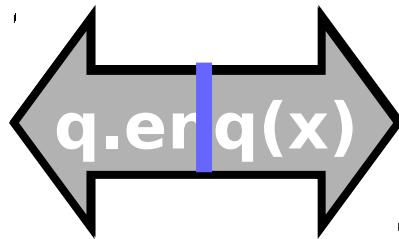
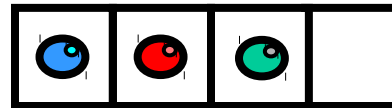


Example



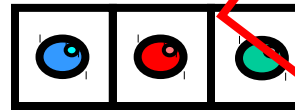


Example

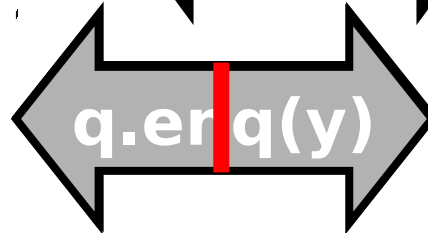
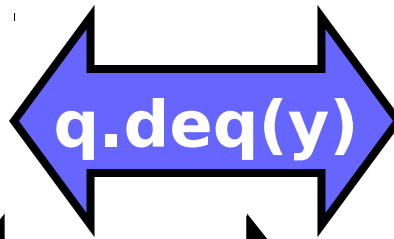
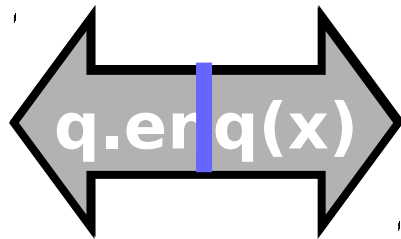




Example

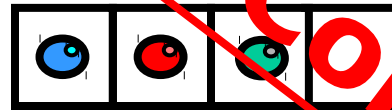


not linearizable

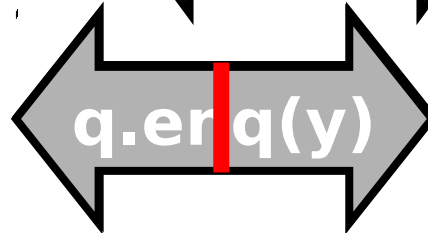
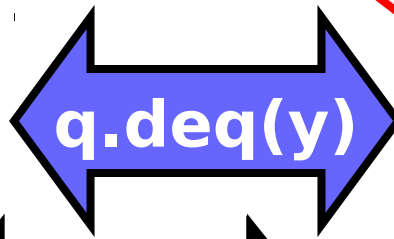
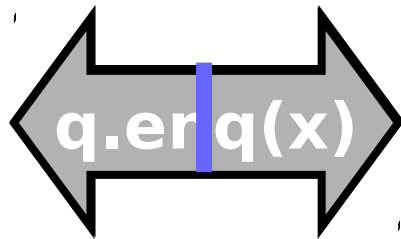




Example



Sequentially Consistent
Yet

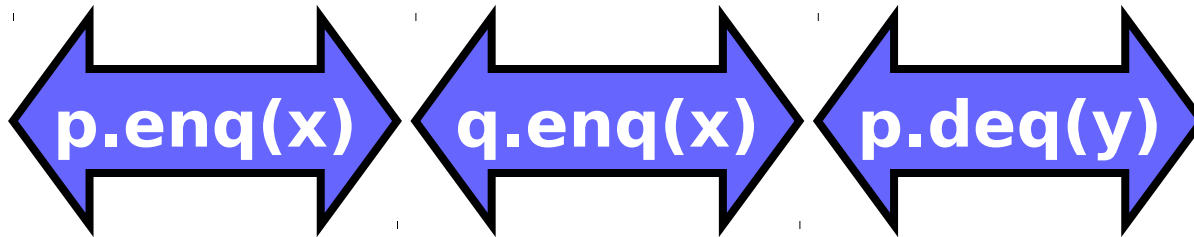


Theorem

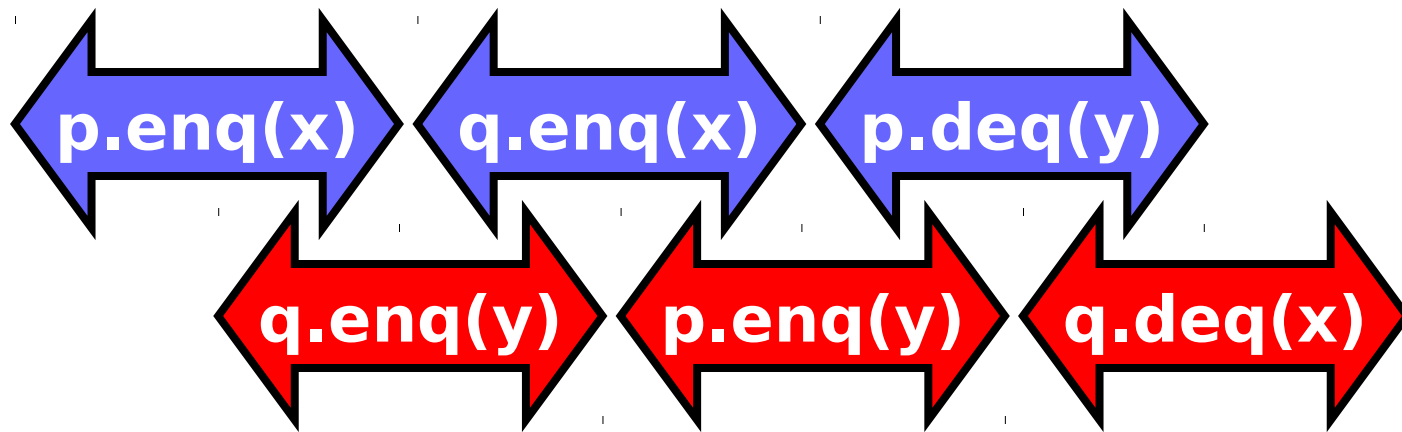
Sequential Consistency is not
a local property

(and thus we lose
composability...)

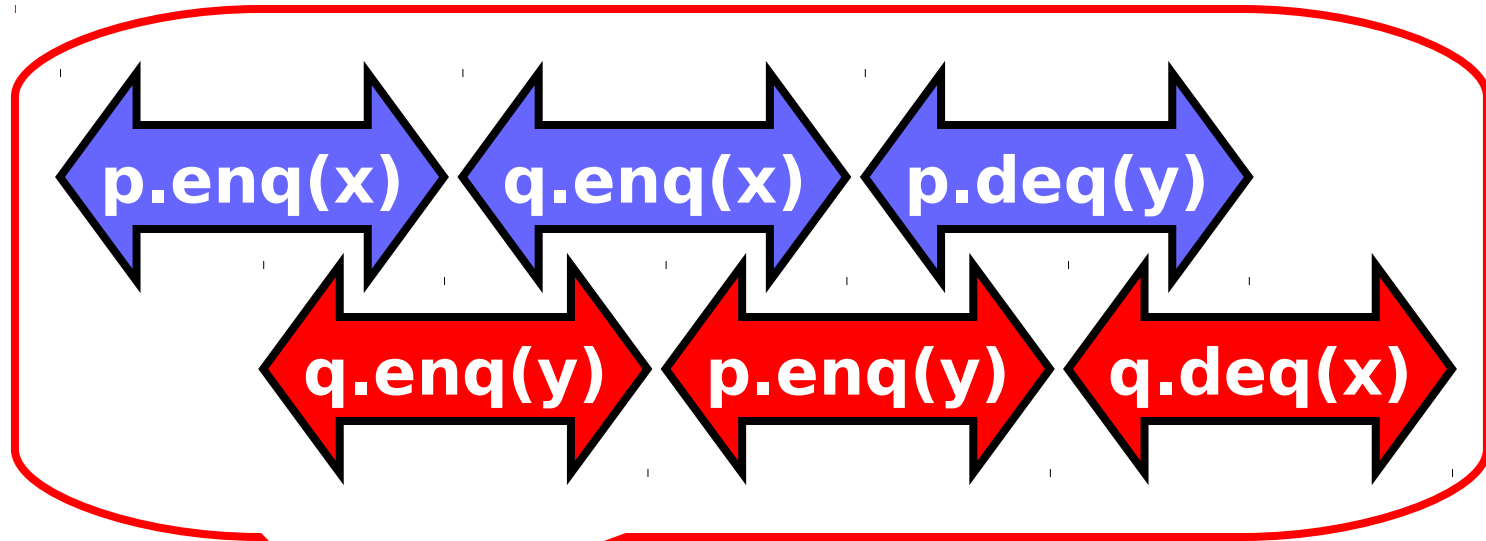
FIFO Queue Example



FIFO Queue Example



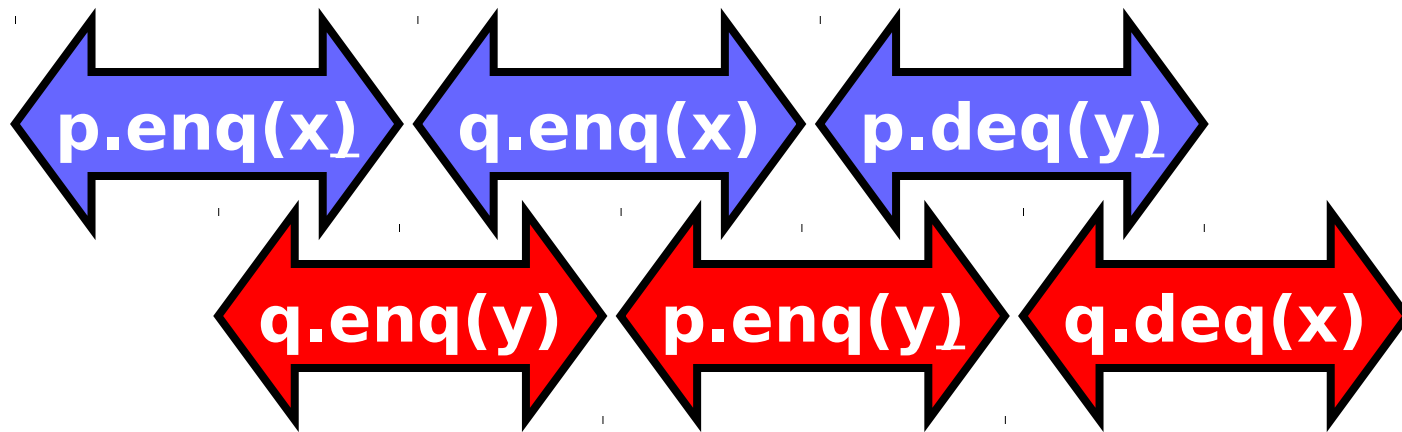
FIFO Queue Example



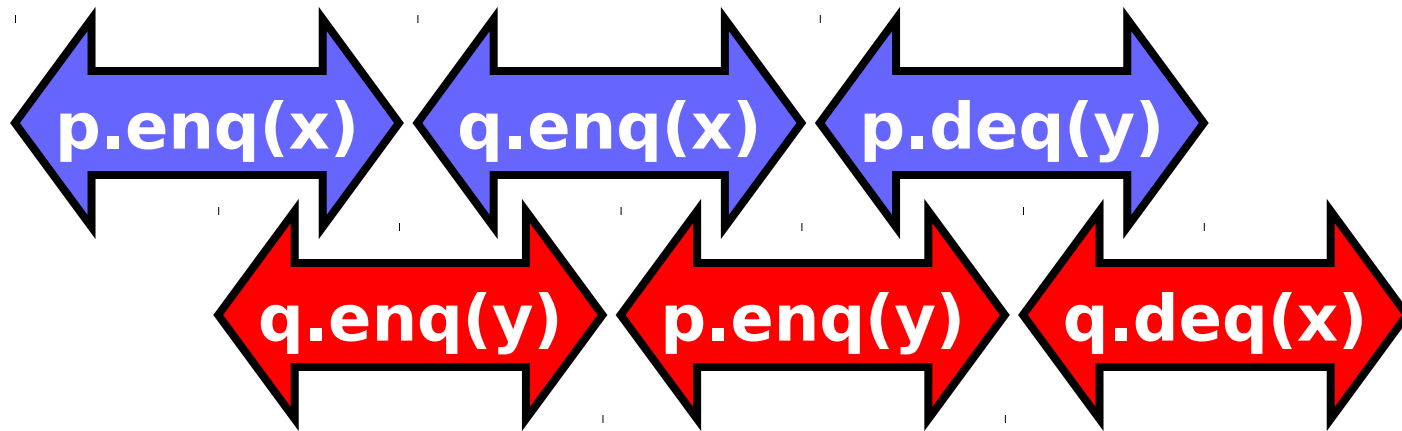
History H



H/p Sequentially Consistent

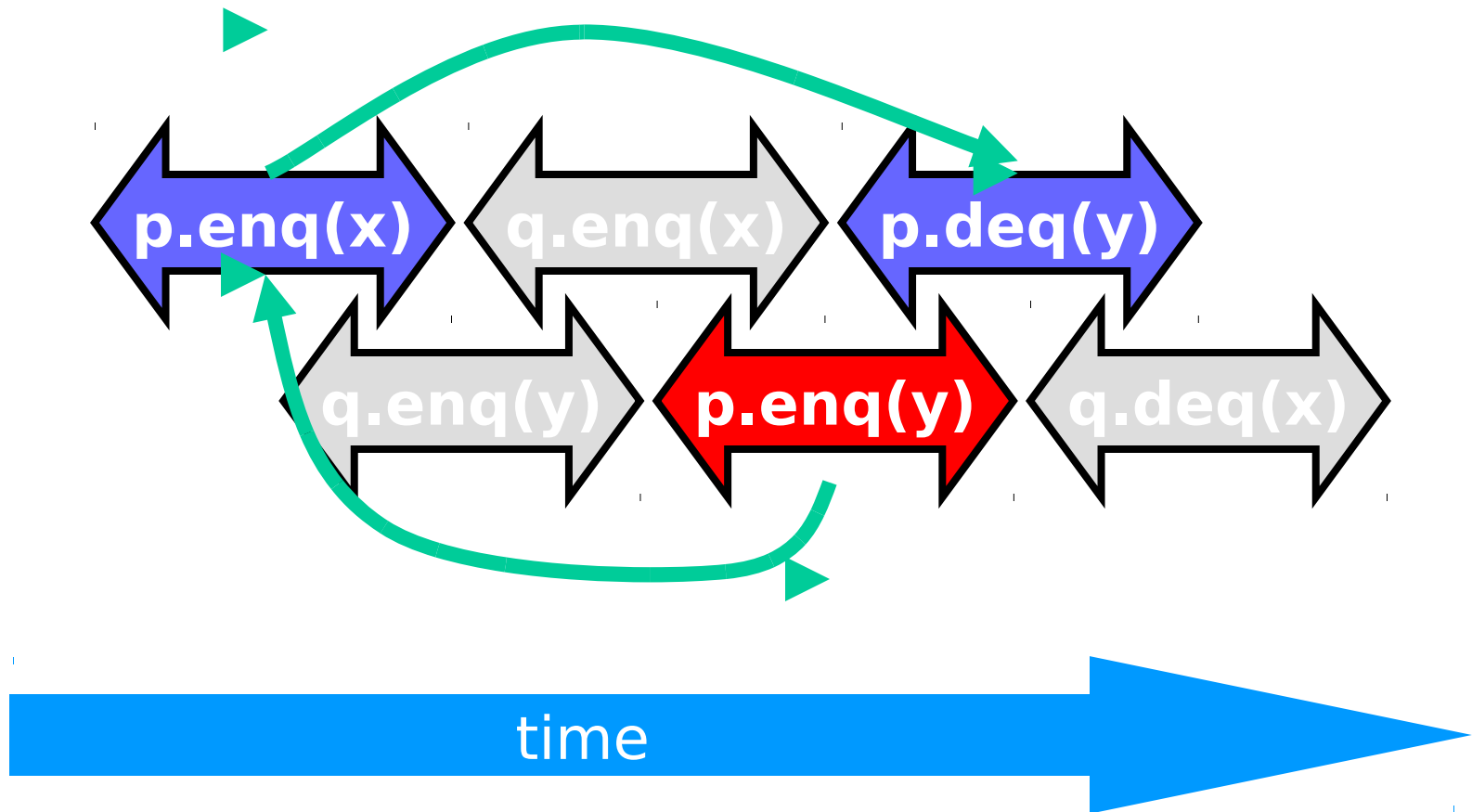


H|q Sequentially Consistent

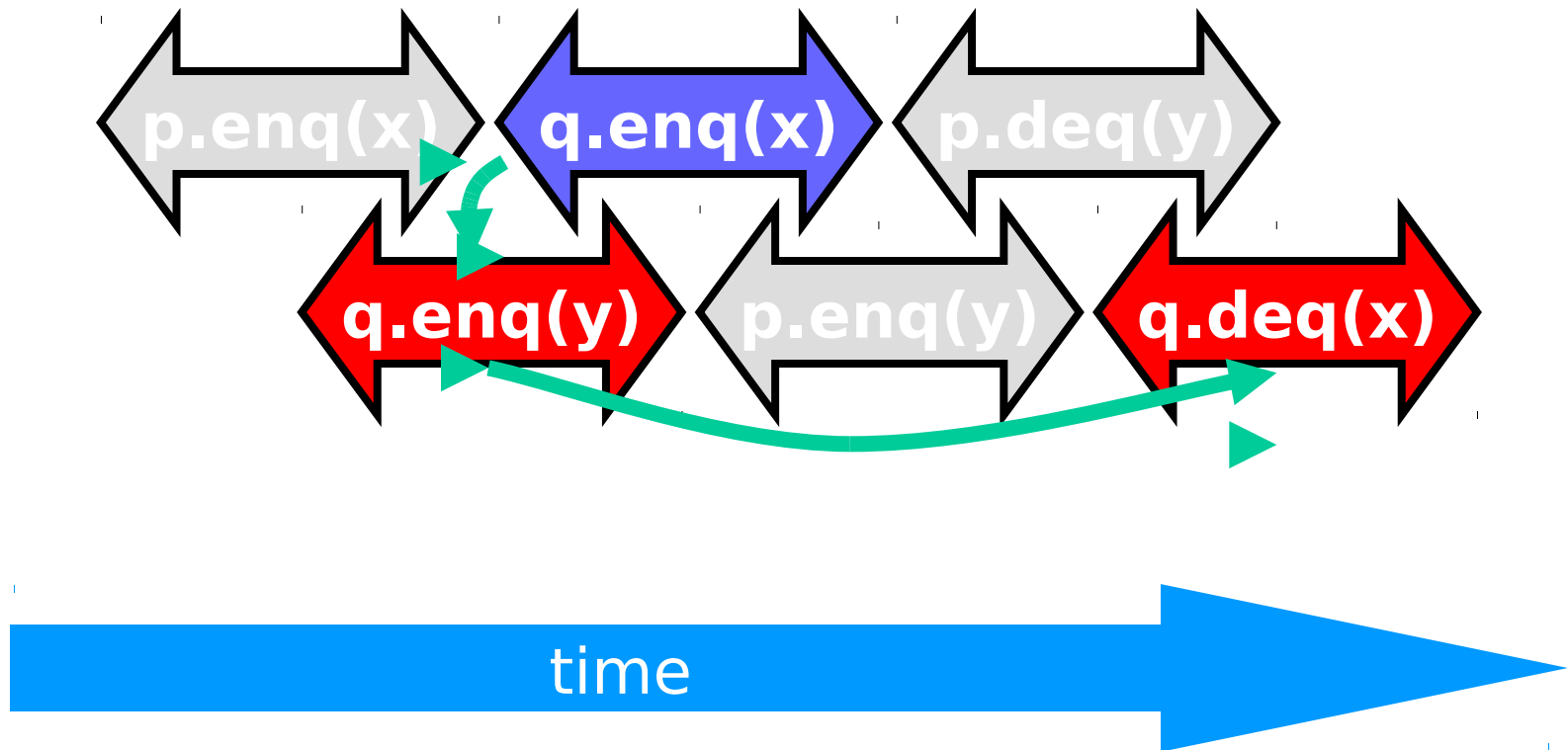


time

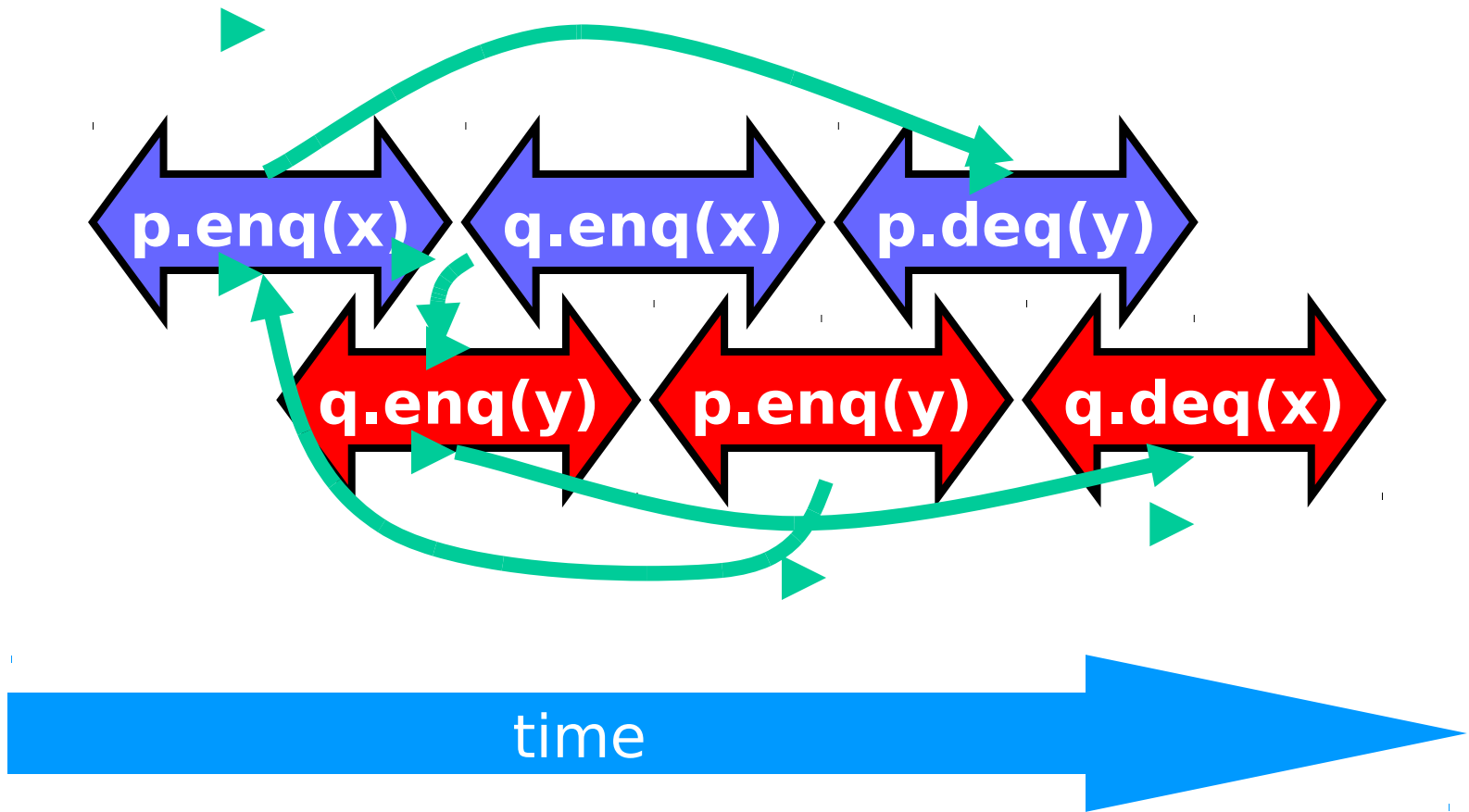
Ordering imposed by p



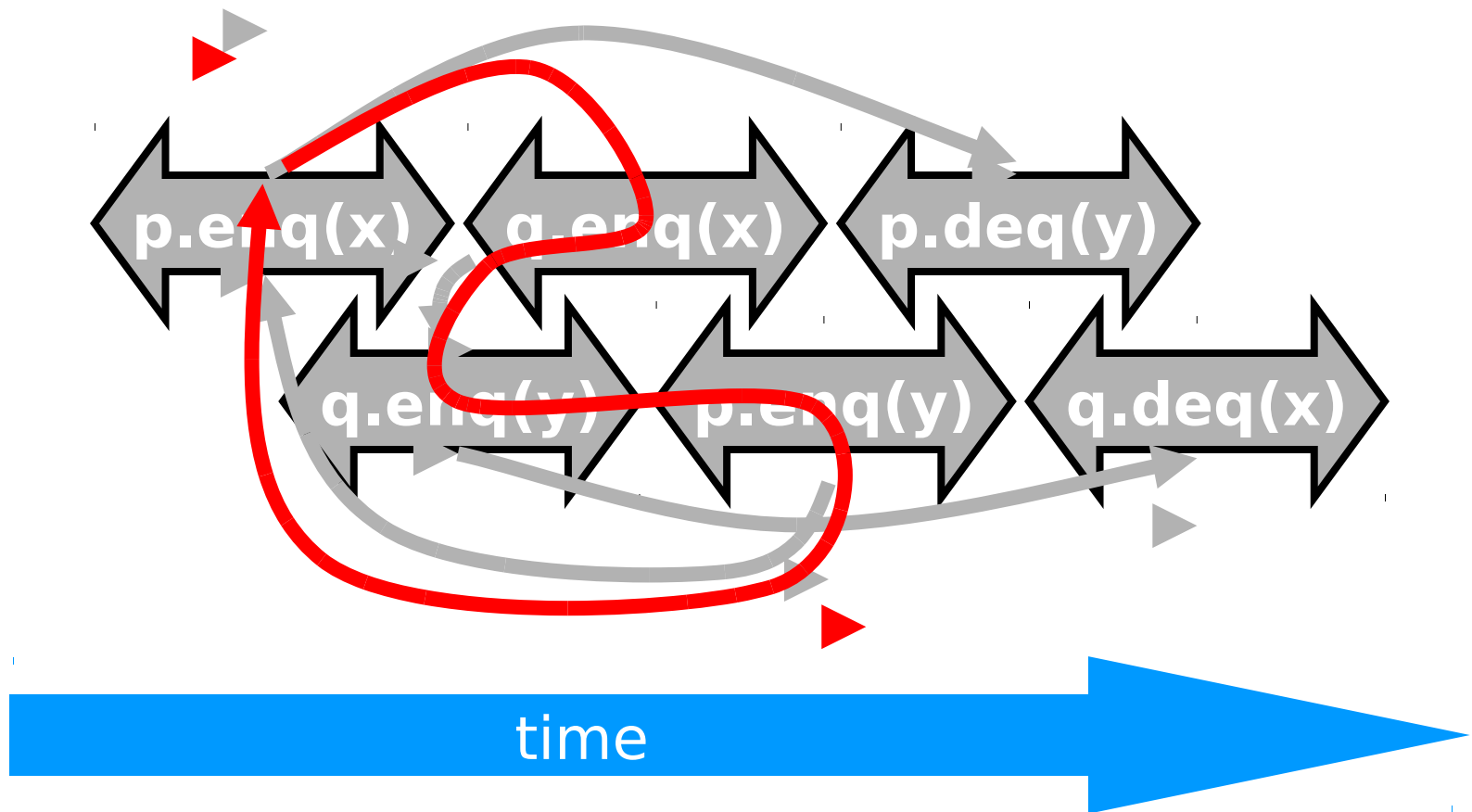
Ordering imposed by q



Ordering imposed by both



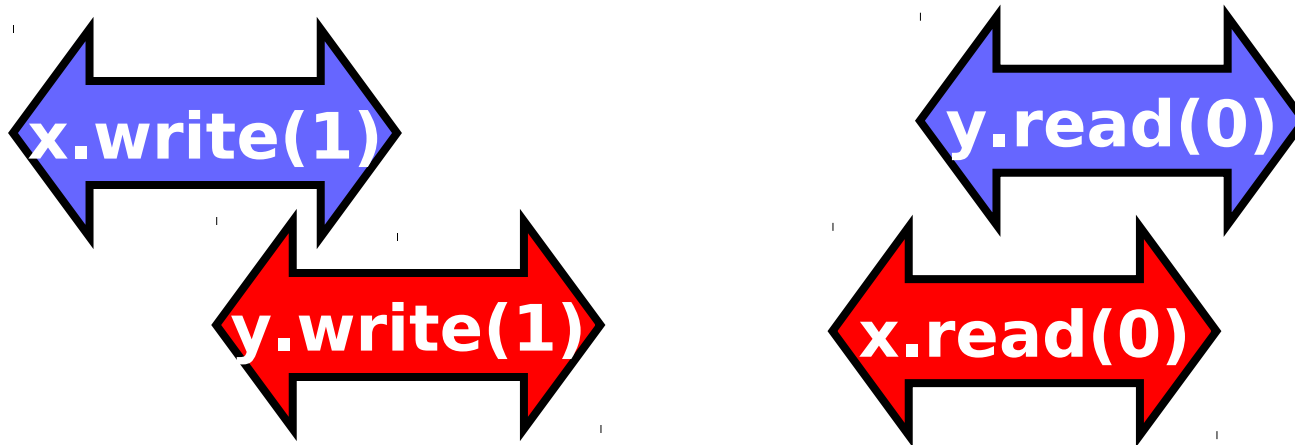
Combining orders



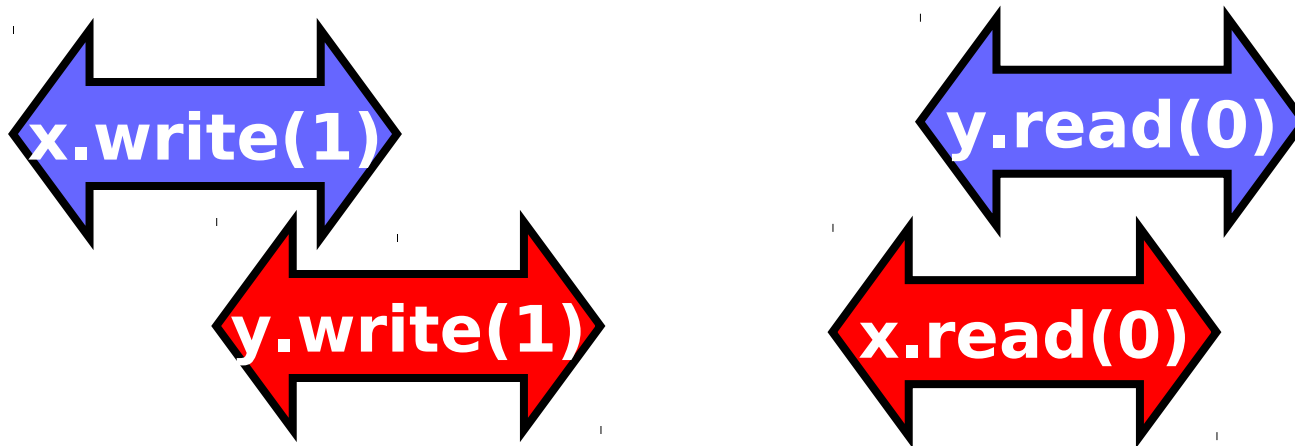
Fact

- Most hardware architectures don't support sequential consistency
- Because they think it's too strong
- Here's another story ...

The Flag Example

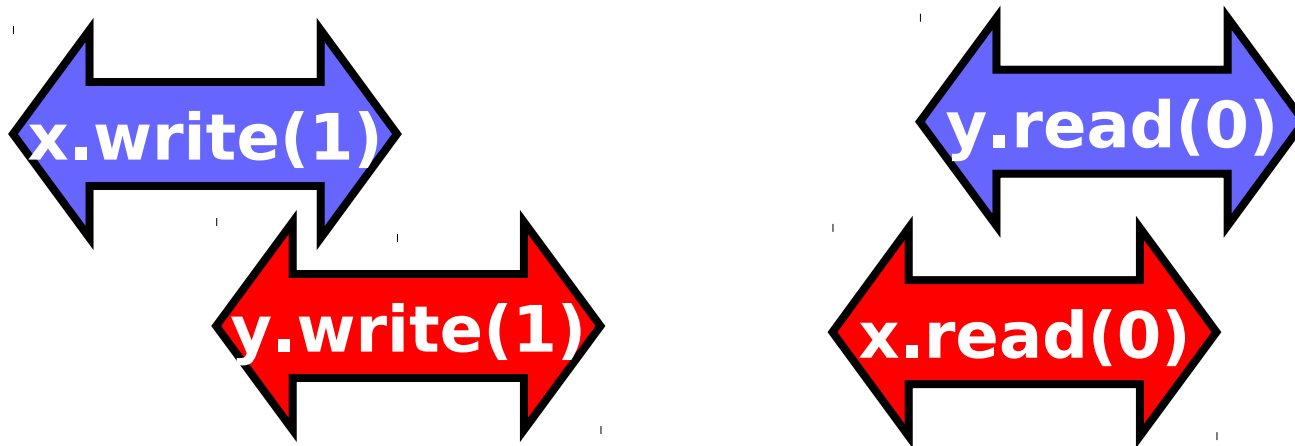


The Flag Example



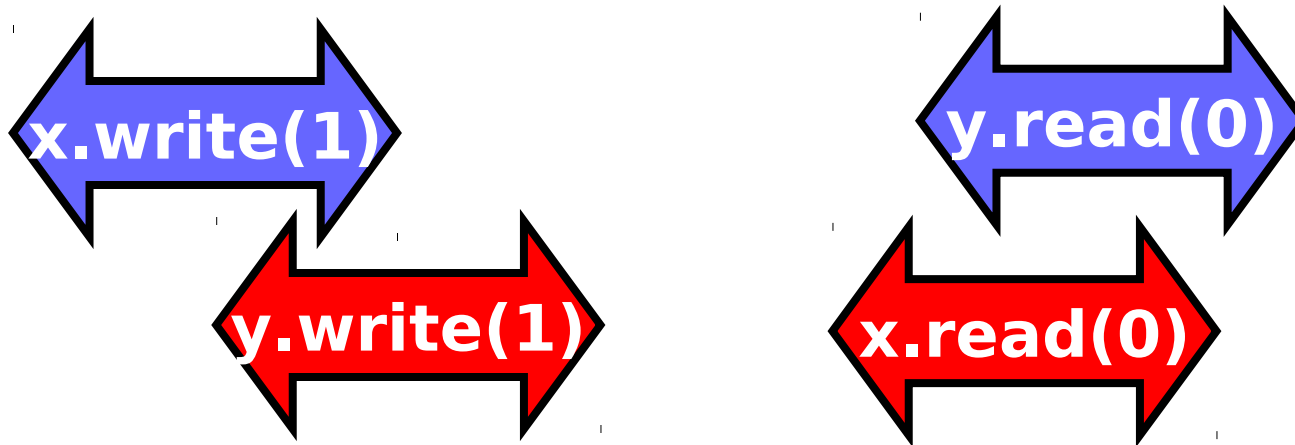
- Each thread's view is sequentially consistent
 - It went first

The Flag Example



- Entire history isn't sequentially consistent
 - Can't both go first

The Flag Example



- Is this behavior really so wrong?
 - We can argue either way ...

Opinion1: It's Wrong

- This pattern
 - Write mine, read yours
- Is exactly the flag principle
 - Beloved of Alice and Bob
 - Heart of mutual exclusion
 - Peterson
 - Bakery, etc.
- It's non-negotiable!

Opinion2: But It Feels So ... Right

- Many hardware architects think that sequential consistency is too strong
- Too expensive to implement in modern hardware
- OK if flag principle
 - violated by default
 - Honored by explicit request

Memory Hierarchy

- On modern multiprocessors, processors do not read and write directly to memory.
- Memory accesses are very slow compared to processor speeds,
- Instead, each processor reads and writes directly to a cache

Memory Operations

- To read a memory location,
 - load data into cache.
- To write a memory location
 - update cached copy,
 - Lazily write cached data back to memory

While Writing to Memory

- A processor can execute hundreds, or even thousands of instructions
- Why delay on every memory write?
- Instead, write back in parallel with rest of the program.

Revisionist History

- Flag violation history is actually OK
 - processors delay writing to memory
 - Until after reads have been issued.
- Otherwise unacceptable delay between read and write instructions.
- Who knew you wanted to synchronize?

Who knew you wanted to ?synchronize

- Writing to memory = mailing a letter
- Vast majority of reads & writes
 - Not for synchronization
 - No need to idle waiting for post office
- If you want to synchronize
 - Announce it explicitly
 - Pay for it only when you need it

Explicit Synchronization

- Memory barrier instruction
 - Flush unwritten caches
 - Bring caches up to date
- Compilers often do this for you
 - Entering and leaving critical sections
- Expensive

Volatile

- In Java, can ask compiler to keep a variable up-to-date with `volatile` keyword
- Also inhibits reordering, removing from loops, & other “optimizations”

Real-World Hardware Memory

- Weaker than sequential consistency
- But you can get sequential consistency at a price
- OK for expert, tricky stuff
 - assembly language, device drivers, etc.
- Linearizability more appropriate for high-level software

Critical Sections

- Easy way to implement linearizability
 - Take sequential object
 - Make each method a critical section
- Problems
 - Blocking
 - No concurrency

Linearizability

- Linearizability
 - Operation takes effect instantaneously between invocation and response
 - Uses sequential specification, locality implies composability
 - Good for high level objects

Correctness: Linearizability

- Sequential Consistency
 - Not composable
 - Harder to work with
 - Good way to think about hardware models
- We will use *linearizability* as in the remainder of this course unless stated otherwise

Progress

- We saw an implementation whose methods were lock-based (deadlock-free)
- We saw an implementation whose methods did not use locks (lock-free)
- How do they relate?

Maximal vs. Minimal

- **Minimal progress:** in some suffix of H, some pending active invocation has a response (some method call eventually completes).
- **Maximal progress:** in every suffix of H, every pending active invocation has a response (every method call always completes).



Progress Conditions

- *Deadlock-free*: some thread trying to acquire the lock eventually succeeds.
- *Starvation-free*: every thread trying to acquire the lock eventually succeeds.
- *Lock-free*: some thread calling a method eventually returns.
- *Wait-free*: every thread calling a method eventually returns.

Progress Conditions

	Non-Blocking	Blocking
Everyone makes progress	Wait-free	Starvation-free
Someone makes progress	Lock-free	Deadlock-free

Summary

- We will look at *linearizable blocking* and *non-blocking* implementations of objects.

Fair Histories

- A history is *fair* if each thread always continues takes steps.
- On multiprocessors this is controlled by the operating system...
- So fair histories are ones in which the operating system guarantees each thread continues to take steps

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