

The Coq Proof Assistant Introduction

Albert-Ludwigs-Universität Freiburg



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- Which semester?
- Experience:
 - Logic courses, Th. comp. science
 - Verification, Hoare Calculus
 - Functional Programming
 - Formal Systems
- Coq:
 - Proof Assistant
 - Programming language
 - (show live)

Software Foundations (Benjamin Pierce et al.)

- Self study course
 - Chapters: Commented source code with exercises
 - <http://www.cis.upenn.edu/~bcpierce/sf/>
Version 2013-07-18
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- Work the chapters at home
 - Meeting once a week for questions/discussion
 - Exercises may be submitted
 - Course Homepage:
<http://proglang.informatik.uni-freiburg.de/teaching/coq-practicum/2014>

- Chapter Exercises
 - Edited versions on course website
 - (* EXPECTED *) Exercise is **strongly** recommended
 - (* NO SOLUTION *) Solution on demand
 - Sample solution 1-2 weeks later
- Graded Exercises
 - **4 graded exercises**, distributed throughout the semester
 - Each 25% of final grade
 - 2 weeks time to submit



Department of Programming Languages
Building 079, Rooms 00-013 and 00-014

- Prof. Dr. Peter Thiemann
- Luminous Fennell:
`fennell@informatik.uni-freiburg.de`

<http://coq.inria.fr/>

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File Edit Options Buffers Tools Coq Proof-General Holes Help
-----
Tactic Notation "aevalR_cases" tactic(first) ident(c) :=
  first;
  [ Case_aux c "E_ANum" | Case_aux c "E_APlus"
  | Case_aux c "E_AMinus" | Case_aux c "E_AMult" ].

(** It is straightforward to prove that the relational and functional
definitions of evaluation agree on all possible arithmetic
expressions... *)

Theorem aeval_iff_aevalR : forall a n,
  (a || n) <-> aeval a = n.
Proof.
  split.
  Case "->".
  intros H.
  aevalR_cases (induction H) SCases; simpl.
  SCases "E_ANum".
    reflexivity.
  SCases "E_APlus".
    rewrite IHaevalR1. rewrite IHaevalR2. reflexivity.
  SCases "E_AMinus".
    rewrite IHaevalR1. rewrite IHaevalR2. reflexivity.
  SCases "E_AMult".
    rewrite IHaevalR1. rewrite IHaevalR2. reflexivity.
  Case "<-".
  generalize dependent n.
  aexp_cases (induction a) SCases;
  simpl; intros; subst.
-----
Imp.v      35% (685,29) <N>  Git-master (Coq Script(4) Holes)--1:55PM-----
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H0 : e2 || n2
IHaevalR1 : aeval e1 = n1
IHaevalR2 : aeval e2 = n2
=====
| n1 + aeval e2 = n1 + n2
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subgoal 2 is:
aeval e1 - aeval e2 = n1 - n2
-----
U:%%- *goals*      36% (14,0) <N>  (Coq Goals Undo.Tree)--1:55PM-----
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Informal

“Clearly, zero is the smallest natural number!”

Formal (Coq)

```
Inductive nat : Set :=
| 0 : nat
| S : nat -> nat.
Inductive le : nat -> nat -> Prop :=
| le_n : forall n : nat, le n n
| le_S : forall n1 n2 : nat,
  le n1 n2 -> le n1 (S n2).
```

```
Theorem le_nat_total: forall n : nat, le 0 n.
Proof. intros n. induction n as [| n'].
(* Case n = 0 *)
apply le_n.
(* Case n = S n' *)
apply le_S. apply IHn'.
Qed.

(* Or with automation *)
Theorem le_nat_total: forall n : nat, le 0 n.
Proof. intros n; induction n as [| n']; auto.
Qed.
```

While Programs

$$e ::= k \mid \text{True} \mid \text{False} \mid x \mid e + e \mid e - e$$
$$s ::= x := e \mid s; s \mid \text{IF } e \text{ THEN } s \text{ ELSE } s \mid \text{WHILE } e \text{ DO } s$$

Lambda Calculus

$$e ::= k \mid \text{True} \mid \text{False} \mid x \mid \text{IF } e \text{ THEN } e \text{ ELSE } e$$
$$\mid \lambda x. e \mid e e$$

- Precise definition of semantics
- Type systems
- Proving properties about programs (e.g. Correctness)