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## Functional Programming

<https://proglang.informatik.uni-freiburg.de/teaching/functional-programming/2022/>

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## Exercise Sheet 12

### 1 Monad Transformers

The `mtl` package builds upon the `transformers` library used in last week's exercise sheet. Instead of programming against concrete a concrete transformer stack the programmer specifies the function's requirements in the type signature. The consumer is then able to choose the underlying monad stack themselves. We can extend last week's table by an "interface" column.

Standard Monad	Transformer	Interface	Base Type	Combined Type
Maybe	<code>MaybeT</code>		Maybe a	m (Maybe a)
Either	<code>ExceptT</code>	<code>MonadError</code>	Either e a	m (Either e a)
Reader	<code>ReaderT</code>	<code>MonadReader</code>	r -> a	r -> m a
Writer	<code>WriterT</code>	<code>MonadWriter</code>	(a, w)	m (a, w)
State	<code>StateT</code>	<code>MonadState</code>	s -> (a, s)	s -> m (a, s)
	<code>RWST</code>	<code>MonadRWS</code>	combines Reader, Writer, State	

#### Exercise 1 (File system state)

The goal of this exercise is to write a monad transformer which implements the `MonadState` interface by writing the state to the file system instead of, like `StateT`, keeping the state in the program memory and passing it from one action to the next.

1. Define a monad transformer `FileStateT` including the `run...` function. The path from which state is read/to which it is written should not be hardcoded but specified as an argument. Implement the customary typeclasses (e.g. `Functor`, ..., `MonadIO`, `MonadTrans`).
2. Implement the `MonadState` interface. All types which implement `Read` and `Show` should be supported.

**Note** Due to Haskell's laziness you will have to use `System.IO.readFile'` for reading the file instead of `readFile` from the `Prelude`. A `get` would otherwise lock the file until the returned value has been forced. An expression such as `get <* put 1` would result in a run-time error.

3. Your transformer should support the other interfaces (`MonadReader`, etc.) if the transformed monad supports these. Write corresponding instances.
4. Write a function `memoizingFib :: MonadState (Map Integer Integer)` to calculate the  $n$ -th `Fibonacci number`. Use the `MonadState` constraint to memoize intermediate results.
5. Given `memoizingFib` two different interpretations. One using the ordinary state transformer, and one using this exercise's `FileStateT`. The latter should check if the file storage exists and initialize it to the empty map if not.

Table 1: Stack calculator operations

Operation	Description
Noop	Leaves the stack unchanged
Pop	Discards the stack's top element
Push $v$	Put's the value $v$ on top of the stack
Dup	Duplicates the topmost value
Dip $p$	Executes $p$ without the topmost value on the stack
Swap	Swaps the two topmost values
Add	Performs the arithmetic operation
Neg	
LessEq	Performs the comparison operation
Not	Performs the logical operation
And	
$p1 :& p2$	Sequences programs $p1$ and $p2$
If $pT pF$	Executes $pT$ if <b>True</b> is on top of the stack and $pF$ otherwise
While $p$	Executes $p$ repeatedly as long as it leaves <b>True</b> on top of the stack

## 2 GADTs

### Exercise 2 (Type safe stack calculator)

We previously implemented a simple stack calculator. It only supported arithmetic operations and always returned 0 on underflow. We now want to extend it with logical operations and disallow programs which underflow the stack. Additionally, there should be no coercion between integer values and booleans.

1. Table 1 lists the operations we want to support. Define the data type **SProg** to represent programs consisting of these operations. Use a GADT to ensure type safety. More specifically, the state of the stack before and after the operation should be tracked in the type.
2. Implement a tag-free interpreter for **SProg**.
3. Define an expression **dup2** to duplicate the *two* values on top of the stack using only the basic operations from Table 1. Use it to write an **SProg** expression to calculate the maximum of two numbers. Write a property test.