
Essentials of Programming Languages

<http://proglang.informatik.uni-freiburg.de/teaching/konzepte/2015/>

Exercise Sheet 5

5.1 Functions

In this exercise we want to add functions to our small language.

$$\langle exp \rangle ::= \dots \mid \mathbf{function} \langle ident \rangle \Rightarrow \langle exp \rangle \mid \langle exp \rangle (\langle exp \rangle)$$

Implement functions as described in the lecture into your interpreter. A good test if your code works properly is the following example:

$$\mathbf{let} \ x = 4 \ \mathbf{in} \ \mathbf{let} \ f = \mathbf{function} \ y \vdash \ x + y \ \mathbf{in} \ \mathbf{let} \ x = 1 \ \mathbf{in} \ f(2)$$

The result should be $Num(6)$ and not $Num(3)$.

5.2 Reduction

The notation of free and bound variables for the lambda calculus

$$e ::= x \mid \lambda x.e \mid e \ e$$

was introduced.

- Reduce the term $(\lambda x.(\lambda y.(\lambda x.x \ y) \ y) \ x) \ x$ using α -, β -, and η -reduction. Show each reduction step and annotate what reduction you used.
- Reduce the term $(\lambda y.(\lambda x.(\lambda y.x \ y)) \ (y \ y))$ using α -, β -, and η -reduction. Show each reduction step and annotate what reduction you used.

5.3 Type derivation

In the lecture, the following inference rules were given:

$$\frac{}{\Gamma \vdash n : \text{Int}}$$
$$\frac{\Gamma \vdash e : \text{Int} \quad \Gamma \vdash e' : \text{Int}}{\Gamma \vdash e + e' : \text{Int}}$$
$$\frac{\Gamma \vdash e_1 : \text{Int} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{if } e_1 \ e_2 \ e_3 : \tau}$$
$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$
$$\frac{\Gamma, x : \tau' \vdash e : \tau''}{\Gamma \vdash \lambda x e : \tau' \rightarrow \tau''}$$
$$\frac{\Gamma \vdash e : \tau' \rightarrow \tau \quad \Gamma \vdash e' : \tau'}{\Gamma \vdash e(e') : \tau}$$

Using these inference rules, you saw a type derivation for $y : \text{Int} \vdash (\lambda x x)y$. Now, give a type derivation for the following judgements, if possible. If not possible, please explain why.

1. $n : \text{Int} \vdash \text{if } (n + 2) (\lambda x x) (\lambda y 1) : \tau \rightarrow \text{Int}$
2. $n : \text{Int} \vdash \text{if } (n) (\lambda x x) (n+1) : \tau \rightarrow \text{Int}$

Submission

Deadline The submission deadline is **05.06.2015, 12:00 (noon)**. Late submissions will not be accepted. Submit your solution to the subversion repository.

Submission Your solution will consist of one *folder* (**exercise5**) for each exercise sheet. Submit one *pdf* file (**<name>_exercise5_<nr>.pdf**) and one *rkt* file (**<name>_exercise5_<nr>.rkt**) per exercise.

Your solution may be either in German or in English. Clear and understandable style is required. You are strongly encouraged to test your solution. Your code must compile without errors (which did not necessarily mean that everything has to work). Provide your source code with comments to understand the intention.