Lecture: Program analysis
Exercise 2
http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2010ss/

Exercise 1: Complete lattices

1. Let $M = \{a, b, c\}$. Define a relation R such that (M, R) is a complete lattice.
2. For a totally ordered set S, $(\mathcal{P}(S), \subseteq)$ is a complete lattice. Define another relation R such that $(\mathcal{P}(S), R)$ is a complete lattice.
3. Is $(\mathbb{R}, \leq)$ a complete lattice? If not, how can you extend $\mathbb{R}$ such that it becomes a complete lattice?
4. Let $|$ be the relation of divisibility, i.e. $a \mid b$ means a divides b. Is
   - $(\mathbb{N}, |)$
   - $(\mathbb{N} \backslash \{0\}, |)$
   - $(\mathbb{N} \backslash \{0\} \cup \{\infty\}, |)$

   a complete lattice?

Exercise 2: Comparing different approaches

Consider the following WHILE program from the slides:

$[y := x]^1$;
$[z := 1]^2$;
`while` $[y > 0]^3$ `do`
  $[z := z * y]^4$;
  $[y := y - 1]^5$;
$[y := 0]^6$

Let $F : (\mathcal{P}(\mathbf{Var} \times \mathbf{Lab}))^{12} \rightarrow (\mathcal{P}(\mathbf{Var} \times \mathbf{Lab}))^{12}$ be the function defined by the data flow equations (cf. slides on p. 31 ff.). Further, let (α, γ) be the Galois connection for the Reaching Definitions analysis (cf. slides on p. 69 ff.)

1. Prove that $\vec{\alpha} \circ G \circ \vec{\gamma} \sqsubseteq F$, i.e. show that

   $$\alpha(G_j(\gamma(RD_1), \ldots, \gamma(RD_{12}))) \subseteq F_j(RD_1, \ldots, RD_{12})$$

   holds for all j. Here, $\vec{f}$ denotes the application of function f to all entries of a tuple or vector.
2. Check whether $F = \vec{\alpha} \circ G \circ \vec{\gamma}$.
3. Prove by induction over n that $(\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset) \sqsubseteq F^n(\emptyset)$.
4. Prove that $\vec{\alpha}(G^n(\emptyset)) \sqsubseteq (\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset)$. You may use that $\vec{\alpha}(\emptyset) = \emptyset$ and $G \sqsubseteq G \circ \vec{\gamma} \circ \vec{\alpha}$.

Submission

- Deadline: 10.05.2010, 12:00, per mail to `bieniusa@informatik.uni-freiburg.de`, or on paper to Annette Bieniusa, Geb. 079, Room 000-14.
- Late submissions will not be marked.
- Do not forget to put your name on the exercise sheet.
- You might want to read up in Appendix A of *Principles of Program Analysis*.