Lecture: Program analysis
Exercise 1
http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2010ss/

1 Data flow analysis: Reaching definitions

Consider the following program written in the WHILE language:

```
x := 1;
y := 1;
r := x;
while (n > 2) do (
    r := x + y;
    x := y;
    y := r;
    n := n - 1
)
```

1. For an input n, what does the program calculate in r?
2. Specify the data flow equations for the program, i.e. for each program point i specify $\mathbf{RD}_\circ(i)$ and $\mathbf{RD}_\bullet(i)$ as on the slides (p. 27 ff.).
3. Calculate the reaching definitions analysis for the program. You can check your solution with the PAG online tool (http://pag.cs.uni-sb.de/).

Solution

1. It calculates the n^{th} Fibonacci number.
2. Let **Lab** be the set of all labels in the program, ? denotes the unknown label.

$$\begin{aligned}
\mathbf{RD}_\bullet(1) &= (\mathbf{RD}_\circ(1)\backslash\{(x,l) \mid l \in \mathbf{Lab}\}) \cup \{(x,1)\}\\
\mathbf{RD}_\bullet(2) &= (\mathbf{RD}_\circ(2)\backslash\{(y,l) \mid l \in \mathbf{Lab}\}) \cup \{(y,2)\}\\
\mathbf{RD}_\bullet(3) &= (\mathbf{RD}_\circ(3)\backslash\{(r,l) \mid l \in \mathbf{Lab}\}) \cup \{(r,3)\}\\
\mathbf{RD}_\bullet(4) &= \mathbf{RD}_\circ(4)\\
\mathbf{RD}_\bullet(5) &= (\mathbf{RD}_\circ(5)\backslash\{(r,l) \mid l \in \mathbf{Lab}\}) \cup \{(r,5)\}\\
\mathbf{RD}_\bullet(6) &= (\mathbf{RD}_\circ(6)\backslash\{(x,l) \mid l \in \mathbf{Lab}\}) \cup \{(x,6)\}\\
\mathbf{RD}_\bullet(7) &= (\mathbf{RD}_\circ(7)\backslash\{(y,l) \mid l \in \mathbf{Lab}\}) \cup \{(y,7)\}\\
\mathbf{RD}_\bullet(8) &= (\mathbf{RD}_\circ(8)\backslash\{(n,l) \mid l \in \mathbf{Lab}\}) \cup \{(n,8)\}\\
\mathbf{RD}_\circ(1) &= \{(x,?),(y,?),(n,?),(r,?)\}\\
\mathbf{RD}_\circ(2) &= \mathbf{RD}_\bullet(1) \qquad \mathbf{RD}_\circ(3) = \mathbf{RD}_\bullet(2)\\
\mathbf{RD}_\circ(4) &= \mathbf{RD}_\bullet(3) \cup \mathbf{RD}_\bullet(8)\\
\mathbf{RD}_\circ(5) &= \mathbf{RD}_\bullet(4) \qquad \mathbf{RD}_\circ(6) = \mathbf{RD}_\bullet(5)\\
\mathbf{RD}_\circ(7) &= \mathbf{RD}_\bullet(6) \qquad \mathbf{RD}_\circ(8) = \mathbf{RD}_\bullet(7)
\end{aligned}$$

3. The solution is given by:

l	$\mathbf{RD}_\circ(l)$	$\mathbf{RD}_\bullet(l)$
1	(x,?),(y,?),(n,?),(r,?)	(x,1),(y,?),(n,?),(r,?)
2	(x,1),(y,?),(n,?),(r,?)	(x,1),(y,2),(n,?),(r,?)
3	(x,1),(y,2),(n,?),(r,?)	(x,1),(y,2),(n,?),(r,3)
4	(x,1),(x,6),(y,2),(y,7),(n,?),(n,8),(r,3),(r,5)	$\mathbf{RD}_\circ(4)$
5	$\mathbf{RD}_\bullet(4)$	(x,1),(x,6),(y,2),(y,7),(n,?),(n,8),(r,5)
6	(x,1),(x,6),(y,2),(y,7),(n,?),(n,8),(r,5)	(x,6),(y,2),(y,7),(n,?),(n,8),(r,5)
7	(x,6),(y,2),(y,7),(n,?),(n,8),(r,5)	(x,6),(y,7),(n,?),(n,8),(r,5)
8	(x,6),(y,7),(n,?),(n,8),(r,5)	(x,6),(y,7),(n,8),(r,5)

2 Constraint based analysis: Control flow analysis

Consider the following program written in a functional language:

$$[[\texttt{fn}\ z \Rightarrow [z]^1]^2 \quad [\texttt{fn}\ y \Rightarrow [y]^3]^4\ ]^5$$

1. What is the result of evaluating this expression?
2. Specify a constraint system for the program, i.e. for each label l specify $C(l)$, and for each variable x, specify $R(x)$ as on the slides (p. 45 ff.).
3. Can you give a solution for the constraint system? Is it a least solution?

Solution

1. The identity function $\texttt{fn}\ y \Rightarrow y$.
2. Constraints relating the values of function abstraction to their labels:

$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(4)$$

Constraints relating the values of variables to their labels:

$$R(z) \subseteq C(1)$$
$$R(y) \subseteq C(3)$$

Conditional constraints induced by function application:

$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2) \Rightarrow C(4) \subseteq R(z)$$
$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2) \Rightarrow C(1) \subseteq C(5)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(2) \Rightarrow C(4) \subseteq R(y)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(2) \Rightarrow C(3) \subseteq C(5)$$

3. The least solution is given by these equations:

$$C(1) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$C(2) = \{\texttt{fn}\ z \Rightarrow z\}$$
$$C(3) = \emptyset$$
$$C(4) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$C(5) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$R(z) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$R(y) = \emptyset$$