Lecture: Program analysis
Exercise 5
http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2010ss/

1 Monotone Frameworks

1. Show that Constant Propagation (as defined in Sec. 2.3.3 of Nielson&Nielson and on the slides) is a Monotone Framework.
2. Show that the Reaching Definitions Analysis is a Bit Vector Framework.

Solution

1. We have to show that
   - $L = ((\mathbf{Var}_* \to \Sigma^\top)_\bot, \sqsubseteq)$ is a complete lattice which satisfies the Ascending Chain Condition, and
   - $\mathcal{F}_{CP} = \{f \mid f \text{ is a monotone function on } \widehat{\mathbf{State}_{\mathbf{CP}}}\}$ contains the identity function and is closed under function composition.

   As defined in chap. 2.3.3., L is by construction a complete lattice. It also satisfies ACC because $\mathbf{Var}_*$ is finite for a given program. Further, the identity function is monotone, and compositions of monotone functions are again monotone.

2. We have to show that
   - $L = (\mathcal{P}(D), \sqsubseteq)$ for a finite set D, and $\sqsubseteq$ is either $\subseteq$ or $\supseteq$, and
   - $\mathcal{F} = \{f : \mathcal{P}(D) \to \mathcal{P}(D) \mid \exists Y_f^1, Y_f^2 : \forall Y \subseteq D : f(Y) = (Y \cap Y_F^1) \cup Y_F^2\}$.

   For the RD Analysis, we have $L = (\mathcal{P}(\mathbf{Var}_* \times \mathbf{Lab}_*^?), \subseteq)$, and $\mathbf{Var}_* \times \mathbf{Lab}_*^?$ is finite. Further, set $Y_f^1 = D \setminus l_k$ and $Y_f^2 = l_g$. Then,

$$\begin{aligned} f(l) &= (l \cap (D \setminus l_k)) \cup l_g \\ &= ((l \setminus l_k) \cap D) \cup l_g \\ &= (l \setminus l_k) \cup l_g \end{aligned}$$

2 Detection of Signs Analysis

In a Detection of Signs Analysis, one models all negative numbers by the symbol $-$, zero by 0, and all positive numbers by $+$. E.g., the set $\{-2, -1, 1\}$ is modeled by $\{+, -\}$.

Let S_* be a program, $\mathbf{Var}_*$ the finite set of variables in S_*. Take $L = \mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\})$ and define an instance of a Monotone Framework for performing Detection of Signs Analysis.

Similarly, take $L' = \mathbf{Var}_* \times \mathcal{P}(\{-, +, 0\})$ and define an instance of a Monotone Framework for performing Detection of Signs Analysis. Is there any difference in the precision between the two approaches?

Solution

The monotone framework is given by the lattice $L = \mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\})$ and the following instantiations:

- $l_1 \sqsubseteq l_2$ iff $\forall x \in \mathbf{Var}_* :\ l_1(x) \subseteq l_2(x)$
- $\bot \in \mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\})$, $\bot(x) = \emptyset \quad \forall x \in \mathbf{Var}_*$
- $\top \in \mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\})$, $\top(x) = \{-, +, 0\} \quad \forall x \in \mathbf{Var}_*$
- $l_1 \sqcup l_2 \in \mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\})$, $(l_1 \sqcup l_2)(x) = l_1(x) \cup l_2(x)$

- $\mathcal{F} = \{f : L \to L \mid f \text{ monotone}\}$
- $F = flow(S_\star)$
- $E = \{init(S_\star)\}$
- $\iota = \bot$
- Let $A_{VA} : \mathbf{AExp} \to \widehat{\mathbf{State}}_{\mathbf{VA}} \to \mathcal{P}(\{-, 0, +\})$ be the function that calculates the sign of an expression using the information in $\widehat{\sigma}$. Then, $f_{\cdot}^{AV}$ is defined by:

$$\begin{array}{lllll} [x := a]^l : & f_l^{AV}(\widehat{\sigma}) & = & \widehat{\sigma}[x \to A_{VA}[\![a]\!]\widehat{\sigma}\,] \\ [\texttt{skip}]^l : & f_l^{AV}(\widehat{\sigma}) & = & \widehat{\sigma} \\ [b]^l : & f_l^{AV}(\widehat{\sigma}) & = & \widehat{\sigma} \end{array}$$

There is no difference in precision for these approaches.