Prof. P. Thiemann, M. Geffken Summer Term 2014

**Static Program Analysis**

http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/

## Exercise Sheet 4

22.05.2014

**Exercise 1** (Complete lattices)

1. Let $M = \{a, b, c\}$. Define a relation $R$ such that $(M, R)$ is a complete lattice.

2. For a totally ordered set $S$, $(\mathcal{P}(S), \subseteq)$ is a complete lattice. Define another relation $R$ such that $(\mathcal{P}(S), R)$ is a complete lattice.

3. Is $(\mathbb{R}, \leq)$ a complete lattice? If not, how can you extend $\mathbb{R}$ such that it becomes a complete lattice?

4. In Exercise 1 on Exercise Sheet 1 you have used the powerset lattice $L_{\mathcal{P}} = (\mathcal{P}(\mathbf{Var}_\star \times \{-, 0, +\}), \subseteq)$ in the Detection of Signs Analysis. Alternatively, you could have used the lattice $L = (\mathbf{Var}_* \to \mathcal{P}(\{-, +, 0\}), \sqsubseteq)$.
   - Provide the definition of the appropriate relation $\sqsubseteq$.
   - What are the values of $\top$ and $\bot$ in $L$?
   - What is $l_1 \sqcup l_2$?
   - Is there any difference in the precision between the two approaches?

**Definitions**

1. A complete partial order $(M, \leq)$ has a *flat* ordering iff

$$\forall x, y \in M : x \leq y \Rightarrow x = \bot \vee x = y$$

2. Let $(M, \leq)$ and $(N, \leq)$ be pointed complete partial orders, and $f : M \to N$. $f$ is
   a) *monotone* iff $x \leq y \Rightarrow f(x) \leq f(y)$;
   b) *strict* iff $f(\bot) = \bot$.

3. Let $(M, \leq)$ and $(N, \leq)$ be complete lattices, and $f : M \to N$. $f$ is *(Scott) continuous* iff $f$ preserves least upper bounds of chains, i.e. for all chains it holds that

$$f\left(\bigsqcup_{i \in I} x^{(i)}\right) = \bigsqcup_{i \in I} f(x^{(i)})$$

**Exercise 2**

Given functions $f : M \to N$ and $g : N \to P$, which of the following statements are true? Give a proof or a counter example.

For pointed complete partial orders $(M, \leq)$ and $(N, \leq)$:

1. If $(N, \leq)$ has a flat ordering and $f$ is monotone, then $f$ is strict or constant.

2. If $(M, \leq)$ has a flat ordering and $f$ is strict, then $f$ is monotone.

For complete lattices $(M, \leq), (N, \leq)$, and $(P, \leq)$:

3. If $(M, \leq)$ satisfies the Ascending Chain Condition and $f$ is monotone, then $f$ is continuous.
4. If $f$ is monotone, then $f$ is strict.
5. If $f$ and $g$ are monotone (continuous, strict), then $g \circ f$ is monotone (continuous, strict).
6. If $f$ is monotone and $\langle x^{(i)} \rangle_{i \in I}$ is a chain in $M$, then $\bigsqcup_{i \in I} f(x^{(i)}) \leq f(\bigsqcup_{i \in I} x^{(i)})$.
7. If $f$ is continuous, then $f$ is also monotone.

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**Submission** In PDF format via email to `geffken AT informatik.uni-freiburg.de`. Please name your single file with the scheme: `ex04-name.pdf`.

- Deadline: **05.06.2014, 12:00**
- Late submissions will not be marked.
- Do not forget to write your name on the exercise sheet.