Prof. P. Thiemann, M. Geffken Summer Term 2014

Static Program Analysis

http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/

Exercise Sheet 5

05.06.2014

Definitions

1. Let $(M, \leq)$ and $(N, \leq)$ be complete lattices, and $f : M \to N$. f is *(Scott) continuous* iff f preserves least upper bounds of chains, i.e. for all chains it holds that
$$f\left(\bigsqcup_{i \in I} x^{(i)}\right) = \bigsqcup_{i \in I} f(x^{(i)})$$
2. Let $(M, \leq)$ be a complete lattice, and $P : M \to \mathbb{B} = \{\texttt{true}, \texttt{false}\}$ a predicate. P is *continuous* iff for every chain $\langle x^{(i)} \rangle_{i \in I}$ in M it holds that $P(x^{(i)}) = \texttt{true}$ for all $i \in I$ implies $P(\bigsqcup_{i \in I} x^{(i)}) = \texttt{true}$.

Exercise 1

Let $(M, \leq)$ be a complete lattice, $f : M \to M$ a continuous function, and $P : M \to \mathbb{B}$ a continuous predicate. Prove that

$$P(\bot) = \texttt{true} \land \forall x \in M : (P(x) = \texttt{true} \Rightarrow P(f(x)) = \texttt{true})$$

implies

$$P(\mathit{lfp}(f)) = \texttt{true}$$

where $\mathit{lfp}(f)$ is the smallest fixed point of f.

Exercise 2 (Galois connections)

Let $(A, \leq)$ and $(G, \leq)$ be partial orders, and (α, γ) be a Galois connection between A and G, i.e. for $X \in G$ and $Y \in A$ it holds:

$$X \leq \gamma(Y) \iff \alpha(X) \leq Y$$

Which of the following statements are true? Give a proof or a counter example.

1. α monotone
2. γ monotone
3. $\alpha = \alpha \circ \gamma \circ \alpha$
4. $\gamma = \gamma \circ \alpha \circ \gamma$

Exercise 3

Let $(L, \leq)$ be a complete lattice, and $f : L \to L$ a monotone function. If $(L, \leq)$ satisfies the ascending chain condition (ACC), then

$$\mathit{lfp}(f) = \bigsqcup_{n} f^{(n)}(\bot)$$

Exercise 4 (Comparing different approaches)
Consider the following WHILE program from the slides:

$[y := x]^1;$
$[z := 1]^2;$
`while` $[y > 0]^3$ `do`
  $[z := z * y]^4;$
  $[y := y - 1]^5;$
$[y := 0]^6$

Let $F : (\mathcal{P}(\mathbf{Var} \times \mathbf{Lab}))^{12} \to (\mathcal{P}(\mathbf{Var} \times \mathbf{Lab}))^{12}$ be the function defined by the data flow equations (cf. slides on p. 31 ff.). Further, let (α, γ) be the Galois connection for the Reaching Definitions analysis (cf. slides on p. 69 ff.)

1. Prove that $\vec{\alpha} \circ G \circ \vec{\gamma} \sqsubseteq F$, i.e. show that

$$\alpha(G_j(\gamma(RD_1), \ldots, \gamma(RD_{12}))) \subseteq F_j(RD_1, \ldots, RD_{12})$$

   holds for all j. Here, $\vec{f}$ denotes the application of function f to all entries of a tuple or vector.

2. Check whether $F = \vec{\alpha} \circ G \circ \vec{\gamma}$.

3. Prove by induction over n that $(\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset) \sqsubseteq F^n(\emptyset)$.

4. Prove that $\vec{\alpha}(G^n(\emptyset)) \sqsubseteq (\vec{\alpha} \circ G \circ \vec{\gamma})^n(\emptyset)$. You may use that $\vec{\alpha}(\emptyset) = \emptyset$ and $G \sqsubseteq G \circ \vec{\gamma} \circ \vec{\alpha}$.

Submission In PDF format via email to `geffken` AT `informatik.uni-freiburg.de`. Please name your single file with the scheme: `ex05-name.pdf`.

- Deadline: **26.06.2014, 12:00**
- Late submissions will not be marked.
- Do not forget to write your name on the exercise sheet.