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**Static Program Analysis**

<http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/>

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**Exercise Sheet 8**

10.07.2014

**Exercise 1 (Monotone Frameworks)**

Read up Sec. 2.3 in the Nielson&Nielson book and familiarise yourself with the *Monotone Frameworks*.

1. Show that Constant Propagation (as defined in Sec. 2.3.3 of Nielson&Nielson and on the slides) is a Monotone Framework.
2. A *Bit Vector Framework* is a special instance of a Monotone Framework where
  - $L = (\mathcal{P}(D), \sqsubseteq)$  for some finite set  $D$  and where  $\sqsubseteq$  is either  $\subseteq$  or  $\supseteq$ , and
  - $\mathcal{F} = \{f : \mathcal{P}(D) \rightarrow \mathcal{P}(D) \mid \exists Y_f^1, Y_f^2 \subseteq D : \forall Y \subseteq D : f(Y) = (Y \cap Y_f^1) \cup Y_f^2\}$
  - a) Show that the Reaching Definitions Analysis is a Bit Vector Framework.
  - b) Show that all Bit Vector Frameworks are indeed Distributive Frameworks.

**Exercise 2 (Relations)**

Consider a context free grammar with start symbol  $N$  and productions  $N ::= Zero \mid Succ(N)$ . It can be rephrased as an inductive definition:

$$Zero \in N \quad \frac{n \in N}{Succ(n) \in N}$$

1. What set  $N$  is defined if you interpret the rules inductively? What does a coinductive interpretation yield?
2. Let us now define a relation  $\leq$  on  $N$  in the following way:

$$Zero \leq n \quad \forall n \in S \quad \frac{n \leq m}{Succ(n) \leq Succ(m)}$$

Let  $R = \{(x, y) \mid x, y \in N : x \leq y\} \subseteq N \times N$ .

- Define the generating function  $S : \mathcal{P}(N \times N) \rightarrow \mathcal{P}(N \times N)$  for this relation. Check that  $S$  is a monotone function.
- Can you find a pair  $(x, y)$  such that  $(x, y) \in gfp(S)$ , but  $(x, y) \notin lfp(S)$ ?
- Prove that  $gfp(S)$  is transitive and reflexive.

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**Submission** In PDF format via email to [geffken@informatik.uni-freiburg.de](mailto:geffken@informatik.uni-freiburg.de). Please name your single file with the scheme: `ex08-name.pdf`.

- Deadline: **17.07.2014, 12:00**
- Late submissions will not be marked.
- Do not forget to write your name on the exercise sheet.