Static Program Analysis

http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/

Solution Sheet 1

6.5.2014

Exercise 1 (Data flow analysis: Detection of Signs)
In a Detection of Signs Analysis one models all negative numbers by the symbol $-$, zero by the symbol 0, and all positive numbers by the symbol $+$. As an example the set $\{-2, -1, 1\}$ is modelled by the set $\{-, +\}$, that is an element of the powerset $\mathcal{P}(\{-, 0, +\})$.

Let $S_\star$ be a program and $\mathbf{Var}_\star$ be the finite set of variables in $S_\star$. Take the *property space* used to represent the data flow information to be $\mathcal{P}(\mathbf{Var}_\star \times \{-, 0, +\})$.

Outline the analysis similary to the Reaching Definitions Analysis as presented in the lecture (see `http://www.imm.dtu.dk/~hrni/PPA/slides1.pdf`). Hint: As in the Reaching Definitions Analysis you want to formulate a *may* analysis and thus use the *combination operator* $\cup$ where an elementary block has more than one predecessor. Before you start, answer yourself the following questions.

- Is the analysis a forward or backward analysis?
- What is the initial value at the start of the analysis?

Consider the following program written in the WHILE language:

```
x := 1;
y := 1;
r := x;
while (n > 2) do (
    r := x + y;
    x := y;
    y := r;
    n := n - 1;
 )
```

1. For an input `n`, what does the program calculate in `r`?
2. Specify the data flow equations for the program, i.e. for each program point i specify $\mathbf{DoS}_\circ(i)$ and $\mathbf{DoS}_\bullet(i)$ similar to $\mathbf{RD}_\circ(i)$ and $\mathbf{RD}_\bullet(i)$ as on the slides (p. 27 ff.).
3. Calculate the Detection of Signs Analysis for the program. Where does the analysis result differ from your intuition?

Solution

1. It calculates the n^{th} Fibonacci number.
2. Let **Sign** be the set of all possible signs $\{-, 0, +\}$, $?_X$ denotes the set $\{(X, s) \mid s \in \mathbf{Sign}\}$ representing the unknown sign for variable x. [1]

[1] Here X is a metavariable that can be substituted by an actual program variable.

$$
\begin{aligned}
&\mathbf{DoS}_\bullet(1) = (\mathbf{DoS}_\circ(1) \backslash ?_x) \cup \{(x,+)\}\\
&\mathbf{DoS}_\bullet(2) = (\mathbf{DoS}_\circ(2) \backslash ?_y) \cup \{(y,+)\}\\
&\mathbf{DoS}_\bullet(3) = (\mathbf{DoS}_\circ(3) \backslash ?_r) \cup \{(r,s) \mid (x,s) \in \mathbf{DoS}_\circ(3)\}\\
&\mathbf{DoS}_\bullet(4) = \mathbf{DoS}_\circ(4)\\
&\mathbf{DoS}_\bullet(5) = (\mathbf{DoS}_\circ(5) \backslash ?_r)\\
&\qquad\qquad \cup \{(r,s) \mid (x,s) \in \mathbf{DoS}_\circ(5) \vee (y,s) \in \mathbf{DoS}_\circ(5)\}\\
&\qquad\qquad \cup \{(r,0) \mid ((x,+) \in \mathbf{DoS}_\circ(5) \wedge (y,-) \in \mathbf{DoS}_\circ(5))\\
&\qquad\qquad\quad \vee ((x,-) \in \mathbf{DoS}_\circ(5) \wedge (y,+) \in \mathbf{DoS}_\circ(5))\}\\
&\mathbf{DoS}_\bullet(6) = (\mathbf{DoS}_\circ(6) \backslash ?_x) \cup \{(x,s) \mid (y,s) \in \mathbf{DoS}_\circ(6)\}\\
&\mathbf{DoS}_\bullet(7) = (\mathbf{DoS}_\circ(7) \backslash ?_y) \cup \{(y,s) \mid (r,s) \in \mathbf{DoS}_\circ(7)\}\\
&\mathbf{DoS}_\bullet(8) = (\mathbf{DoS}_\circ(8) \backslash ?_n)\\
&\qquad\qquad \cup \{(n,-) \mid (n,0) \in \mathbf{DoS}_\circ(8) \vee (n,-) \in \mathbf{DoS}_\circ(8)\}\\
&\qquad\qquad \cup \{(n,0) \mid (n,+) \in \mathbf{DoS}_\circ(8)\} \cup \{(n,+) \mid (n,+) \in \mathbf{DoS}_\circ(8)\}\\
&\mathbf{DoS}_\circ(1) = ?_x \cup ?_y \cup ?_n \cup ?_r\\
&\mathbf{DoS}_\circ(2) = \mathbf{DoS}_\bullet(1) \qquad \mathbf{DoS}_\circ(3) = \mathbf{DoS}_\bullet(2)\\
&\mathbf{DoS}_\circ(4) = \mathbf{DoS}_\bullet(3) \cup \mathbf{DoS}_\bullet(8)\\
&\mathbf{DoS}_\circ(5) = \mathbf{DoS}_\bullet(4) \qquad \mathbf{DoS}_\circ(6) = \mathbf{DoS}_\bullet(5)\\
&\mathbf{DoS}_\circ(7) = \mathbf{DoS}_\bullet(6) \qquad \mathbf{DoS}_\circ(8) = \mathbf{DoS}_\bullet(7)
\end{aligned}
$$

3. The solution is given by:

l	$\mathbf{DoS}_\circ(l)$	$\mathbf{DoS}_\bullet(l)$
1	$?_x \cup ?_y \cup ?_n \cup ?_r$	$\{(x,+)\} \cup ?_y \cup ?_n \cup ?_r$
2	$\mathbf{DoS}_\bullet(1)$	$\{(x,+),(y,+)\} \cup ?_n \cup ?_r$
3	$\mathbf{DoS}_\bullet(2)$	$\{(x,+),(y,+),(r,+)\} \cup ?_n$
4	$\{(x,+),(y,+),(r,+)\} \cup ?_n$	$\mathbf{DoS}_\circ(4)$
5	$\mathbf{DoS}_\bullet(4)$	$\{(x,+),(y,+),(r,+)\} \cup ?_n$
6	$\mathbf{DoS}_\bullet(5)$	$\{(x,+),(y,+),(r,+)\} \cup ?_n$
7	$\mathbf{DoS}_\bullet(6)$	$\{(x,+),(y,+),(r,+)\} \cup ?_n$
8	$\mathbf{DoS}_\bullet(7)$	$\{(x,+),(y,+),(r,+)\} \cup ?_n$