Prof. P. Thiemann, M. Geffken Summer Term 2014

Static Program Analysis

http://proglang.informatik.uni-freiburg.de/teaching/programanalysis/2014ss/

Solution Sheet 2

08.05.2014

Exercise 1 (Constraint based analysis: Control flow analysis)
Consider the following program written in a functional language:

$$[[\texttt{fn}\ z \Rightarrow [z]^1]^2 \quad [\texttt{fn}\ y \Rightarrow [y]^3]^4\,]^5$$

1. What is the result of evaluating this expression?
2. Specify a constraint system for the program, i.e. for each label l specify $C(l)$, and for each variable x, specify $R(x)$ as on the slides (p. 45 ff.).
3. Can you give a solution for the constraint system? Is it a least solution?

Solution

1. The identity function $\texttt{fn}\ y \Rightarrow y$.
2. Constraints relating the values of function abstraction to their labels:

$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(4)$$

Constraints relating the values of variables to their labels:

$$R(z) \subseteq C(1)$$
$$R(y) \subseteq C(3)$$

Conditional constraints induced by function application:

$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2) \Rightarrow C(4) \subseteq R(z)$$
$$\{\texttt{fn}\ z \Rightarrow z\} \subseteq C(2) \Rightarrow C(1) \subseteq C(5)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(2) \Rightarrow C(4) \subseteq R(y)$$
$$\{\texttt{fn}\ y \Rightarrow y\} \subseteq C(2) \Rightarrow C(3) \subseteq C(5)$$

3. The least solution is given by these equations:

$$C(1) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$C(2) = \{\texttt{fn}\ z \Rightarrow z\}$$
$$C(3) = \emptyset$$
$$C(4) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$C(5) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$R(z) = \{\texttt{fn}\ y \Rightarrow y\}$$
$$R(y) = \emptyset$$

Exercise 2 (Types)

1. Provide simple typing rules for the following syntactical constructs that could be part of the fun language on the slides.

   a) $$\frac{\cdots}{\Gamma \vdash e_1 \texttt{ + } e_2 :}$$

   b) $$\frac{\cdots}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 :}$$

2. Extend the typing rules such that function application effects are considered (cf. slides on p. 92 ff.).

Solution

1. a) $$\frac{e_1 : \texttt{int} \qquad e_2 : \texttt{int}}{\Gamma \vdash e_1 \texttt{ + } e_2 : \texttt{int}}$$

   b) $$\frac{e_1 : \texttt{bool} \qquad e_2 : T \qquad e_3 : T}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : T}$$

2. a) $$\frac{e_1 : \texttt{int} \,\&\, \varphi_1 \qquad e_2 : \texttt{int} \,\&\, \varphi_2}{\Gamma \vdash e_1 \texttt{ + } e_2 : \texttt{int} \,\&\, \varphi_1 \cup \varphi_2}$$

   b) $$\frac{e_1 : \texttt{bool} \,\&\, \varphi_1 \qquad e_2 : T \,\&\, \varphi_2 \qquad e_3 : T \,\&\, \varphi_3}{\Gamma \vdash \texttt{if } e_1 \texttt{ then } e_2 \texttt{ else } e_3 : T \,\&\, \varphi_1 \cup \varphi_2 \cup \varphi_3}$$